R&D Delegation in a Bertrand Duopoly with Spillovers*

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Abstract

We provide an extensive comparison of three cost-reducing R&D games where duopolistic firms (i) cooperatively conduct in-house R&D, (ii) non-cooperatively choose in-house R&D, and (iii) delegate R&D to an independent profit-maximizing laboratory. Firms are assumed to behave à la Bertrand on the final market. We establish that delegating R&D to an independent laboratory is Pareto optimal when between-firm technological spillovers are not too high and the production of R&D services by the laboratory, for the two firms, is complementary.

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1 Introduction

Cooperative research and development (R&D) and technology outsourcing have received considerable attention in the theoretical industrial organization literature.[1] One stream of that literature considers a monopolistic laboratory which can license a patented process innovation to vertically-related firms by making take-it-or-leave-it offers to downstream firms. Most of these analyses build on Katz and Shapiro's (1986) complete information model where the laboratory incurs no cost (i.e., R&D costs have been paid in a previous period), and each downstream firm is a potential user of one unit of the innovative input. A central feature of extensions to the seminal paper is that they root the inventor's ability to earn benefits in various characterizations of the strategic interaction between potential licensees, who may compete in quantities or in prices on a final product market. In particular, Kamien, Oren and Tauman (1992) compare alternative licensing strategies, namely a fixed fee, a per unit royalty, or the auctioning of a fixed number of licenses. In this paper, it is demonstrated that the results obtained in the Cournot setup are qualitatively the same as in the Bertrand case.

Another research stream pays particular attention to how knowledge externalities affect R&D outcomes, firms' profits, and social welfare. In their seminal analysis, d'Aspremont and Jacquemin (1988) consider a duopoly where firms invest in deterministic cost-reducing R&D, may choose R&D cooperatively, and compete on the goods market *à la* Cournot. They show that cooperation is R&D augmenting and welfare improving when between-firm technological spillovers are sufficiently high. The numerous extensions to their model assume in-house R&D, either in each firm's separate laboratory or in a jointly owned one, with firms sharing the operating costs (see Martin (2001) for an overview of that literature). A few papers have used the same framework while considering Bertrand competition on the goods market. They include a survey by Hinloopen (2000) who describes a model that encompasses most specific forms of the literature, with either quantity and or price competition on the product market. Amir, Evstigneev and Wooders (2003) unify and generalize the results of this literature without relying on specific functional forms, and by removing the separation between Cournot and Bertrand cases. They confirm two central results of this research stream: (i) R&D cooperation increases firms' profits; and (ii) the profitability of R&D cooperation increases with the level of R&D spillovers.

Our starting point is that the technology market is not restricted to either R&D cooperation among competitors, or to purchasing a licensed pre-formatted R&D output. We consider cases in which firms contract with for-profit laboratories to delegate the production of R&D services that fit their particular requirements.[2] This is an accelerating phenomenon, as according to the National Science Foundation (2003) the amount of contracted-out R&D in the United States, in millions of U.S.$, was 6,710 in 1998, 9,240 in 1999, and 14,785 in 2000.[3]

Interestingly, in many instances rivals delegate R&D to a common independent laboratory through bilateral contractual agreements. For example, in the chemical industry Bayer and ICI (two European firms which compete on world markets) signed multi-annual contracts in 1999 and 2000 respectively with Symyx, a U.S.-based private laboratory. The latter receives payments by providing access to a proprietary high speed combinatorial technology toward the production of firm-specific R&D outputs, that is high value speciality polymers. Similarly, in the steel industry ThyssenKrupp and Arcelor (two major European suppliers), bilaterally contracted in 1995 with VAI, a firm which specializes in the design of new steel production methods, for the outsourcing of process R&D services. The objective is to produce wide thin strips of stainless and carbon steels directly from the molten metal, omitting the stages of slab casting and rolling.

Real-world contracts specify the required R&D outcome in exchange of a payment scheme,[4] in relation to detailed non-compete clauses or exclusivity conditions. For instance, such clauses appear in a contract signed in 1997 by Millennium (a U.S.-based private laboratory in the biotechnology sector) and Monsanto (a US provider of agricultural products), for some gene sequencing R&D services. In this contract, the laboratory agrees not to make any other significant agricultural enterprise benefit from the collaboration without the prior written consent of Monsanto for some period of time. Another example is a contract signed in 1998 by the same laboratory and Bayer (a European pharmaceutical company), for the provision of molecules from some genomics technology. It stipulates that the firm may not benefit from the outcome of collaborative research agreements signed in the past by the laboratory with explicitly identified competitors, including Hoffmann-La Roche, Eli Lilly, and Pfizer.[5]

As in Katz and Shapiro (1986), we consider a profit-maximizing laboratory which may serve none, one, or two firms. However, we abandon their assumption that the laboratory is in a monopolistic position inherited from past innovative efforts. Rather, we assume the laboratory responds to payment schemes by providing firm-specific R&D services at some costs. This assumption captures situations where a laboratory derives income from tailor-made R&D which it provides to firms. The latter are in a duopoly and benefit from cost-reducing R&D outputs as in d'Aspremont and Jacquemin's (1988). We assume firms behave *à la* Bertrand on the final market and may delegate R&D non-cooperatively to the laboratory. This allows us to investigate the extent to which the results obtained in the Cournot setup (Vencatachellum and Versaevel 2004) still hold in the Bertrand case.[6]

Our model allows for two R&D externalities: the usual between-firm cost-reducing technological spillovers (*direct* externalities); and positive or negative within-laboratory externalities (*indirect* externalities) if R&D services are respectively complements or substitutes.[7] We use this framework to (i) ask when the laboratory earns positive profits, (ii) compare R&D outcomes, firms' profits, and social welfare in a delegated game with those in non-cooperative and cooperative in-house R&D games, and (iii) derive conditions for R&D delegation to Pareto-dominate non-cooperative and cooperative R&D. We can illustrate our results graphically in the plane of between-firm technological spillovers, and within-laboratory R&D externalities.[8]

Intuitively, substitute or complement R&D projects which are run by laboratory and firm-level spillovers combine to determine the level of competition between the two firms for R&D services.

Whether competition for the laboratory's resources is soft or tight is reflected by each firm's payment offers to the laboratory. These in turn determine the laboratory's ability to earn excess benefits, the equilibrium R&D outcomes, the delegating firms' profits, and social welfare. As a result, we establish that the laboratory earns positive benefits only if its production of firm-specific R&D services are substitutable, or not too complementary, and technological spillovers are sufficiently low. We also show that R&D delegation Pareto-dominates non-cooperation and cooperation if within-laboratory R&D services are sufficiently complementary, and technological spillovers are sufficiently low. Our results are qualitatively similar to those obtained by Vencatachellum and Versaevel (2004) who consider Cournot competition between firms in the final market in the same framework. Hence, this paper demonstrates that previous results obtained in the Cournot setup are robust to allowing for a change in the nature of competition.

The remainder of the paper is as follows. Section 2 presents the three R&D games, defines and discusses the equilibrium concepts. Section 3 establishes that the laboratory maximizes aggregate benefits and establishes conditions under which it earns zero benefits. Then in section 4 we rank the outcomes of the three R&D games as a function of firm-level technological spillovers and within-laboratory spillovers, and illustrate the results graphically in the direct and indirect externalities plane. Next, Section 4.4 investigates whether one of the three games can Pareto-dominate the other two and derives some policy implications. Finally, section 5 concludes. All proofs and Figures are in the Appendix.

2 R&D Games

Following Singh and Vives (1984), consumers are characterized by a quadratic utility function defined over two potentially differentiated products:

$$U(\mathbf{q}) = a(q_1 + q_2) - b(q_1^2 + 2\theta q_1 q_2 + q_2^2), \tag{1}$$

where $\mathbf{q} = (q_1, q_2)$, $\theta \in [0, 1)$, a and b are positive parameters. Maximizing (1) with respect to $\mathbf{q}$, subject to the budget constraint, and simplifying gives the demand function for each good as:

$$q_i(\mathbf{p}) = \frac{1}{(1+\theta)b}\left[a - \frac{p_i}{(1-\theta)} + \frac{\theta p_j}{(1-\theta)}\right], \tag{2}$$

where $\mathbf{p} = (p_1, p_2)$ is the vector of prices, $i, j = 1, 2$, and $i \neq j$.

On the production side, we consider a duopoly where each firm incurs a constant unit cost of production which it can reduce through process innovations. We also assume, as in d'Aspremont and Jacquemin (1988), a unit cost of production:

$$c_i(\mathbf{x}) = c - x_i - \beta x_j, \tag{3}$$

where $\mathbf{x} = (x_1, x_2) \in \Re_+^2$ is the vector of R&D outputs obtained by firms, $\beta \in (0, 1)$ measures technological spillovers, and $c \in (0, a)$ is the marginal cost parameter. It follows that firm i's gross profit equals:

$$\pi_i(\mathbf{p}, \mathbf{x}) = (p_i - c_i(\mathbf{x}))q_i(\mathbf{p}). \tag{4}$$

The next section formalizes three cost-reducing R&D games in extensive forms.

2.1 Cooperative R&D

In a first stage, the duopoly cooperatively chooses in-house R&D outcomes in the two proprietary laboratories by maximizing joint profits. The cost of R&D is given by:

$$r(x_i) = \frac{\gamma}{2} x_i^2, \tag{5}$$

for $i = 1, 2$, and where γ is a positive parameter. In a second stage, given the chosen R&D outcomes each firm non cooperatively maximizes individual profits by choosing its price. In this game, we denote firm i's net profit, as a function of $\mathbf{x}$, by:

$$\pi_i^{\mathrm{c}}(\mathbf{x}) \equiv \pi_i(\mathbf{p}^{\mathrm{c}}, \mathbf{x}) - r(x_i), \tag{6}$$

where $\mathbf{p}^{\mathrm{c}} \equiv (p_1^{\mathrm{c}}(\mathbf{x}), p_2^{\mathrm{c}}(\mathbf{x}))$. Firms' symmetric net equilibrium profits are denoted by π^{c}.

Definition 1 (NE) *The symmetric final market outcome* $\mathbf{p}^{\mathrm{c}}$ *is a Nash equilibrium if:*

$$\pi_i(\mathbf{p}^{\mathrm{c}}, \mathbf{x}) \geq \pi_i(p_i, p_j^{\mathrm{c}}(\mathbf{x}), \mathbf{x}), \tag{7}$$

all $\mathbf{x}$*, all* p_i*,* $i, j = 1, 2$*,* $i \neq j$*.*

Instead of cooperatively choosing their R&D, firms may decide to do so non-cooperatively, as explained below.

2.2 Non-Cooperative R&D

In a first stage, firms non-cooperatively conduct R&D in-house by maximizing their individual profits in their own R&D, with each firm's R&D costs given by (5). The second stage is as in the cooperative

R&D game. In this game, we denote firm i's net profit as a function of $\mathbf{x}$ by:

$$\pi_i^{\mathrm{n}}(\mathbf{x}) \equiv \pi_i(\mathbf{p}^{\mathrm{n}}, \mathbf{x}) - r(x_i), \tag{8}$$

where $\mathbf{p}^{\mathrm{n}} \equiv (p_1^{\mathrm{n}}(\mathbf{x}), p_2^{\mathrm{n}}(\mathbf{x}))$. Each firm's symmetric net equilibrium profits are denoted by π^{n}.

Definition 2 (SPNE) *The symmetric equilibrium prices and in-house* R&D *outcomes* $(\mathbf{p}^{\mathrm{n}}, \mathbf{x}^{\mathrm{n}})$ *are a subgame-perfect Nash equilibrium if:*

i) $\mathbf{p}^{\mathrm{n}}$ *is a* NE *as in Definition 1, and*

ii) $\mathbf{x}^{\mathrm{n}}$ *is a* NE*, that is* $\pi_i^{\mathrm{n}}(\mathbf{x}^{\mathrm{n}}) \geq \pi_i^{\mathrm{n}}(x_i, x_j^{\mathrm{n}})$*, for all* x_i*,* $i, j = 1, 2$*,* $i \neq j$*.*

This game is identical to another one where, in lieu of producing R&D in-house, there are two independent laboratories. In that alternative game, each firm writes a contract with one exclusive laboratory to obtain specific R&D services in exchange of transfer payments. In our complete information setup, it follows that a firm's problem is as in (8), but $r(x_i)$ is now firm i's payment for x_i, and the laboratory earns zero benefits. The problem is however different if there is a unique, common, and independent laboratory, from which the two firms buy R&D services. We tackle this next.

2.3 Delegated R&D

In a first stage, the two firms (principals) simultaneously and non cooperatively purchase x_1 and x_2 by offering contingent transfer payments $t_i(\mathbf{x})$ to one common laboratory (an agent). Firm i's strategy is a transfer function t_i that associates a payment to the laboratory in exchange of any possible choice of cost-reducing R&D services. We denote the set of transfer payments by:

$$T \equiv \{t | t(\mathbf{x}) \geq 0 \quad \text{for all} \quad \mathbf{x}\}. \tag{9}$$

In a second stage, given $(t_1(\mathbf{x}), t_2(\mathbf{x})) \equiv \mathbf{t}$, the laboratory chooses the amounts of firm-specific R&D services, at a cost $s(\mathbf{x})$, that maximize its benefit given by:

$$\mathcal{L}(\mathbf{x}) = t_1(\mathbf{x}) + t_2(\mathbf{x}) - s(\mathbf{x}). \tag{10}$$

We assume that the laboratory may choose to contract with no firm, in which case it earns zero benefits. This leads to a participation constraint:

$$\mathcal{L} \geq 0. \tag{11}$$

When discussing policy implications later, we shall consider situations where (11) holds with strict inequality. This would be the case if the laboratory incurs positive (arbitrarily small) installation costs, or faces a profitable outside option. We denote the set of R&D services which, given strategies $\mathbf{t}$, maximize the laboratory's benefits by:

$$X(\mathbf{t}) \equiv \arg\max_{\mathbf{x}} \mathcal{L}(\mathbf{x}(\mathbf{t})). \tag{12}$$

The third stage is as the final stage in the other two games.

As in the cooperative and non-cooperative cases, information is complete among firms, but the laboratory needs not know downstream cost and demand functions. An outcome of the delegated R&D game is a three-tuple $(\mathbf{x}^{\mathrm{d}}, \mathbf{t}^{\mathrm{d}}, \mathbf{p}^{\mathrm{d}})$, where $\mathbf{x}^{\mathrm{d}}$ denotes the laboratory's equilibrium choice, $\mathbf{t}^{\mathrm{d}}$ firms' equilibrium payments, and $\mathbf{p}^{\mathrm{d}}$ equilibrium prices on the final market. In this game firm i's net profit, as a function of $\mathbf{x}$, equal:

$$\pi_i^{\mathrm{d}}(\mathbf{x}) \equiv \pi_i(\mathbf{p}^{\mathrm{d}}, \mathbf{x}) - t_i^{\mathrm{d}}(\mathbf{x}), \tag{13}$$

where $\mathbf{p}^{\mathrm{d}} \equiv (p_1^{\mathrm{d}}(\mathbf{x}), p_2^{\mathrm{d}}(\mathbf{x}))$.[9] Firms' symmetric net equilibrium profits are denoted by π^{d}.

Definition 3 (TSPNE) *The symmetric equilibrium prices, delegated* R&D *outcomes, and transfer payments* $(\mathbf{x}^{\mathrm{d}}, \mathbf{t}^{\mathrm{d}}, \mathbf{p}^{\mathrm{d}})$ *are a truthful subgame-perfect Nash equilibrium if:*

i) $\mathbf{p}^{\mathrm{d}}$ *is a* NE *as in Definition 1,*

ii) $(\mathbf{x}^{\mathrm{d}}, \mathbf{t}^{\mathrm{d}})$ *is a* NE*, that is* $\mathbf{x}^{\mathrm{d}} \in X(\mathbf{t}^{\mathrm{d}})$ *and there is no* $i = 1, 2$*,* $t_i \in T$*, and no* $\mathbf{x} \in X(t_i, t_j^{\mathrm{d}})$ *such that* $\pi_i^{\mathrm{d}}(\mathbf{x}) > \pi_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}})$*, and*

iii) t_i^{d} *is truthful relative to* $\mathbf{x}^{\mathrm{d}}$*, that is for all* $\mathbf{x}$ *either* $\pi_i^{\mathrm{d}}(\mathbf{x}) = \pi_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}})$*, or* $\pi_i^{\mathrm{d}}(\mathbf{x}) < \pi_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}})$ *and* $t_i^{\mathrm{d}}(\mathbf{x}) = 0$*,* $i, j = 1, 2$*,* $i \neq j$*.*

Intuitively, in any truthful equilibrium, a firm offers a transfer $t_i^{\mathrm{d}}(\mathbf{x})$ that exactly reflects its individual valuation of the laboratory's choice of $\mathbf{x}$ with respect to $\mathbf{x}^{\mathrm{d}}$, all $\mathbf{x}$. Definition 3-iii) refers to two possible cases. Either gross profits $\pi_i(\mathbf{p}^{\mathrm{d}}(\mathbf{x}), \mathbf{x})$ exceed net equilibrium profits $\pi_i(\mathbf{p}^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}}), \mathbf{x}^{\mathrm{d}}) - t_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}})$, and the difference between transfer offers $t_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}})$ and $t_i^{\mathrm{d}}(\mathbf{x})$ is set equal to the difference between gross profits $\pi_i(\mathbf{p}^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}}), \mathbf{x}^{\mathrm{d}})$ and $\pi_i(\mathbf{p}^{\mathrm{d}}(\mathbf{x}), \mathbf{x})$. Or principal i's gross profits with $\mathbf{x}$ are strictly less than net equilibrium profits obtained with $\mathbf{x}^{\mathrm{d}}$, in which case the transfer $t_i^{\mathrm{d}}(\mathbf{x})$ is set to zero.

For this game, as in Laussel and Le Breton (2001), by equilibria we mean truthful subgame-perfect Nash equilibria, and we recall two properties that justify the choice of this solution concept. Firstly, for any set of transfer offers by any one of the two firms, there exists a truthful strategy in the other firm's best-response correspondence. This existence property implies that a firm can restrict itself to truthful strategies. Secondly, all truthful Nash equilibria are coalition-proof. This stability property says that total net profits, as obtained in a truthful subgame-perfect equilibrium by the two firms, are higher than in any other subgame-perfect Nash equilibria. The two properties hold for all given choices of $\mathbf{p}$ in the final stage, including $\mathbf{p}^{\mathrm{d}}$.[10]

For the sake of tractability we specify a laboratory's cost function as follows:

$$s(\mathbf{x}) = \frac{\gamma}{2}(x_1^2 + x_2^2) - \delta x_1 x_2, \tag{14}$$

for $i = 1, 2$ and $i \neq j$, and $\delta \in [-\gamma, \gamma)$ captures complementary (substitutable) R&D services in the laboratory if $\delta > 0$ ($\delta < 0$). If $\delta = 0$, the laboratory is as efficient as each firm's proprietary laboratory. Note that the term $\delta x_1 x_2$ in (14) is the simplest way to capture complementarity or substitutability between two variables. A nice aspect of this formalization is that complementarity or substitutability is reflected by the sign a single parameter as suggested by Milgrom and Roberts (1990, p. 517) in an illustrative example. The same algebraic specification appears in the complete information version of the cost function of a common agent in Martimort and Stole (2001), and in the utility function of a common agent in Martimort and Stole (2003). The existence of within-laboratory spillovers gives rise to indirect externalities, which are defined, and contrasted with between-firm technological spillovers, below.

2.4 Direct and Indirect Externalities

Define firm i's concentrated profits as $\pi_i(\mathbf{x}) \equiv \pi(\mathbf{p}(\mathbf{x}), \mathbf{x})$. In all three games, concentrated profits vary with technological spillovers which are measured by β. These spillovers are a *direct* externality because firm i's gross profits not only depend on x_i, but also on x_j for all $\beta > 0$. These externalities are negative (positive) if an increase in x_j has a negative (positive) impact on firm i's concentrated profits.

Property 1 (Direct Externalities) *For $i, j = 1, 2$, and $i \neq j$*

$$\frac{\mathrm{d}\pi_i(\mathbf{x})}{\mathrm{d}x_j} \begin{array}{c} > \\ = \\ < \end{array} 0 \quad \text{if and only if} \quad \beta \begin{array}{c} > \\ = \\ < \end{array} \widetilde{\beta},$$

where $\widetilde{\beta} \equiv \theta/(2 - \theta^2)$.

In what follows, we identify positive (negative) direct externalities with $\beta > (<) \widetilde{\beta}$. As for *indirect* externalities, they appear only in the delegated R&D game where the laboratory's choice of x_i affects the costs of providing x_j, with $i \neq j$. Indirect externalities are negative (positive) if serving higher quantities to a firm makes it more (less) costly for the laboratory to serve the other one, i.e. if the production of R&D services are substitutable (complementary). More formally:

Property 2 (Indirect Externalities) *For* $i, j = 1, 2,$ *and* $i \neq j$

$$\frac{\mathrm{d}s(\mathbf{x})}{\mathrm{d}x_i\mathrm{d}x_j} \begin{smallmatrix}<\\=\\>\end{smallmatrix} 0 \quad \text{if and only if} \quad \delta \begin{smallmatrix}>\\=\\<\end{smallmatrix} 0.$$

Typically, R&D services are complements (i.e., $\delta > 0$) when the laboratory can serve the two firms by using the same resources. They are substitutes (i.e., $\delta < 0$) when there are bottlenecks in the laboratory's capacity to simultaneously supply the two firm-specific services.

Definition 4 (Pivotal Point) $(\beta, \delta) = (\widetilde{\beta}, 0)$ *is a pivotal point where there are neither direct nor indirect externalities.*

We now establish how the laboratory's choice compares with the cooperative game. We then derive a condition under which the laboratory earns positive benefits. This condition partitions the (β, γ) space, which we refer to as the externalities plane in the remainder of the paper.

3 Profits Maximization and Distribution

Let the aggregate benefits function for the two firms and the laboratory be:

$$\Lambda(\mathbf{x}) = \pi_1^{\mathrm{d}}(\mathbf{x}) + \pi_2^{\mathrm{d}}(\mathbf{x}) - s(\mathbf{x}). \tag{15}$$

Proposition 1 (Joint Profits Maximization) *In all* TSPNE*, the laboratory's choice of* R&D *services to maximize its benefits (10) is equivalent to maximizing aggregate benefits (15).*

Proposition 1 is a restatement of Bernheim and Whinston (1986) adapted to our context. It says that the non-cooperative attempt by firms to maximize individual profits by delegating R&D leads to a choice of $\mathbf{x}$ that maximizes the aggregate benefits of all parties, including the laboratory. By maximizing the sum of the two firms profits, net of R&D costs, the laboratory internalizes both direct and indirect externalities. However, Proposition 1 is silent on consumers' welfare. We will be able to address this issue once we compute the quantities of R&D services produced by the laboratory and compare them to the two other games. This is the subject of section 4.1.

Denote by Λ the maximum aggregate benefits obtained by maximizing (15) with respect to $\mathbf{x}$. The following proposition characterizes the distribution of Λ between the laboratory and the two firms.

Proposition 2 (Joint Profits Distribution) *There exists a continuous strictly decreasing frontier in the externalities plane* (β, δ)*, denoted by* $\delta_{\mathcal{L}=0}$*, and which includes the pivotal point, such that in all TSPNE the laboratory earns positive benefits if* $\delta < \delta_{\mathcal{L}=0}$*, and exactly breaks even otherwise.*

Proposition 2 says that the magnitude of indirect externalities, as captured by δ, for a given value of the direct externalities parameter β, determines the laboratory's ability to appropriate a share of innovation benefits. This is because indirect externalities, in combination with technological spillovers, impact the nature of competition between the two firms on the intermediate market for R&D. This competition is reflected by their offers of transfer payments $(t_1^{\mathrm{d}}(\mathbf{x}), t_2^{\mathrm{d}}(\mathbf{x}))$. On the one hand, if both direct and indirect externalities are negative, a firm's concentrated profits decrease with the other firm's R&D (Property 1), and serving one firm increases the laboratory's cost of serving the other (Property 2). This is a case of tough competition between the two firms for the laboratory's services, which is a source of positive profits for it. On the other hand, if both externalities are positive, a firm's concentrated profits are increasing in the other firm's R&D, and serving one firm decreases the laboratory's cost of serving the other. Thus, competition for the laboratory's resources is relatively soft, and the laboratory earns no benefits. When the externalities are of opposite signs, the laboratory's ability to appropriate benefits depends on their magnitudes. This opposition gives rise to $\delta_{\mathcal{L}=0}$, which can thus be viewed as a weighted sum of direct and indirect externalities.

Propositions 1 and 2 are useful for the comparison of the three R&D games outcomes at the pivotal point.

Proposition 3 (No Externalities) *The outcomes of the three games are the same at the pivotal point.*

At the pivotal point, there are no direct and indirect externalities. This implies that solutions in $\mathbf{x}$ are the same in the three R&D games. In the delegated game the laboratory earns zero benefits, as if firms were relying on in-house R&D capabilities, because the pivotal point is on $\delta_{\mathcal{L}=0}$.

We now solve the three R&D games by backward induction and rank the performance of the three games in the externalities plane. The explicit solutions of the games are in Appendix A.

4 Comparing the Three Games

We partition the externalities plane by deriving frontiers on which R&D, profits, or welfare are equal in the delegated R&D game and in one of the two alternative games. By welfare, we mean the sum of consumer surplus, firms' profits, and the laboratory's benefits. For the sake of completeness, we also include the comparison of the outcomes of the cooperative and non-comparative games. As in d'Aspremont and Jacquemin (1988), firms have full information, which allows us to compare delegated R&D with the non-cooperative and cooperative cases. However, we do not need to extend the complete information assumption to the laboratory. Note from the onset that, Property 3 implies all such frontiers include the pivotal point.

4.1 R&D outcomes

Lemma 1 (Cooperative, Non-Cooperative, and Delegated R&D)

i) There exists a continuous frontier $\delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$ *in the externalities plane such that in all TSPNE* $x^{\mathrm{d}} \gtreqqless x^{\mathrm{c}}$ *if and only if* $\delta \gtreqqless \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$*, with*

$$\begin{array}{lll} \delta_{\mathcal{L}=0} > \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0 & \textit{for} & \beta < \widetilde{\beta}; \\ \delta_{\mathcal{L}=0} = \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0 & \textit{for} & \beta = \widetilde{\beta}; \\ \delta_{\mathcal{L}=0} < \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0 & \textit{for} & \beta > \widetilde{\beta}. \end{array}$$

ii) There exists a continuous frontier $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ *in the externalities plane, such that in all TSPNE* $x^{\mathrm{d}} \gtreqqless x^{\mathrm{n}}$ *if and only if* $\delta \gtreqqless \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$*, with*

$$\begin{array}{lll} 0 < \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} < \delta_{\mathcal{L}=0} & \textit{for} & \beta < \widetilde{\beta}; \\ \delta_{\mathcal{L}=0} = \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} = 0 & \textit{for} & \beta = \widetilde{\beta}; \\ 0 > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} > \delta_{\mathcal{L}=0} & \textit{for} & \beta > \widetilde{\beta}. \end{array}$$

Direct and indirect externalities combine to give Lemma 1. First consider Lemma 1-(i). The cooperative and delegated games yield the same R&D solution when there are no indirect externalities because of Proposition 1 (which says that the laboratory maximizes aggregate benefits in equilibrium), and of Property 2 (which implies that costs are the same in both games when $\delta = 0$). We know that the independent laboratory is more (less) efficient than in-house laboratories when indirect externalities are positive (negative), that is when $\delta > 0$ ($\delta < 0$). This completes the partitioning of the externalities plane for R&D output in the two games under scrutiny.

Second, consider Lemma 1-(*ii*). Recall that, from Property 1, optimal R&D is greater (smaller) in the cooperative than in the non cooperative game for positive (negative) direct externalities. Let direct externalities be positive. If indirect externalities are also positive, the laboratory's higher efficiency means that delegated R&D exceeds the cooperative, and hence the non-cooperative, solutions. If indirect externalities are negative, the laboratory is at a disadvantage in the production of R&D over in-house laboratories. However, as it internalizes inter-firm direct externalities via the transfer payments it receives, it is only for sufficiently negative indirect externalities that non-cooperative R&D exceeds the delegated game solution. Consequently, $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ must cross in the South-East quadrant of the externalities plane.

Now let direct externalities be negative. If indirect externalities are also negative, the laboratory's lower efficiency than in-house laboratories means that the delegated solution is smaller than the cooperative, and by transitivity of the non-cooperative one also. However, as the laboratory gains in efficiency as δ increases, there exist sufficiently high positive indirect externalities for the R&D outcome under the delegated game to exceed that under the non-cooperative game. Hence $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ must lie in the North-West quadrant of the externalities plane.

[**Insert figure 1 about here**]

The juxtaposition of $\delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$ and $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ in the externalities plane, as illustrated in Figure 1, allows us to rank optimal R&D across the three games. It is of interest that optimal R&D in the delegated game is greater than in either of the two games for sufficiently high indirect externalities, even when direct externalities are negative. This result stands in contrast with cooperative R&D always being less than non-cooperative one for negative direct externalities.

4.2 Firms' Profits

Lemma 2 (Cooperative, Non-Cooperative, and Delegated Profits)

i) There exists a continuous frontier $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ *in the externalities plane such that in all TSPNE* $\pi^{\mathrm{d}} \gtreqqless \pi^{\mathrm{c}}$ *if and only if* $\delta \gtreqqless \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$*, with*

$$\begin{array}{lll} 0 < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} < \delta_{\mathcal{L}=0} & \textit{for} & \beta < \widetilde{\beta}; \\ \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} = \delta_{\mathcal{L}=0} = 0 & \textit{for} & \beta = \widetilde{\beta}; \\ 0 = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} > \delta_{\mathcal{L}=0} & \textit{for} & \beta > \widetilde{\beta}. \end{array}$$

ii) There exists a continuous frontier $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ *in the externalities plane such that in all TSPNE* $\pi^{\mathrm{d}} \gtreqqless \pi^{\mathrm{n}}$ *if and only if* $\delta \gtreqqless \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$*, with*

$$\begin{array}{lll} 0 < \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} & \textit{for} & \beta < \widetilde{\beta}; \\ \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} = 0 & \textit{for} & \beta = \widetilde{\beta}; \\ 0 = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} > \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} & \textit{for} & \beta > \widetilde{\beta}. \end{array}$$

The intuition for $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ follows also from how the two externalities combine. For the same reasons as in Section 4.1, aggregate benefits are *ceteris paribus* increasing in indirect externalities. However, when part of the aggregate benefits accrue to the laboratory, which is the case for $\delta < \delta_{\mathcal{L}=0}$, then indirect externalities must be sufficiently positive to generate enough surplus to compensate for the laboratory's benefits. Hence, if direct externalities are negative, the locus which equalizes firms' profits in the delegated and cooperative games must lie in the North-West quadrant of the externalities plane.

It cannot however lie above $\delta_{\mathcal{L}=0}$ where aggregate benefits in the delegated game exceed those in the cooperative game, but are divided equally between the two firms. If direct externalities are positive, the frontier is confounded with $\delta = 0$ because of Proposition 1, the cost structure being the same in both games and the laboratory earning zero benefits.

The intuition for the $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ locus is as follows. Recall that a firm's profits in the cooperative game always exceed those under the non-cooperative one because cooperation internalizes direct externalities and prevents R&D duplication. As a firm's profits in both the cooperative and delegated games are equal along $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$, by transitivity delegated profits exceed non cooperative ones along that locus. Consider negative direct externalities. For $\delta = 0$, along that line cooperative profits are greater than those obtained in the delegated game. However, firms' profits in the delegated game are increasing in indirect externalities (Lemma D-2 in Appendix D). Hence, there exists a unique decreasing continuous locus in the North-West quadrant of Figure 2 such that $\pi^{\mathrm{d}} = \pi^{\mathrm{n}}$.

By the same token, there must exist a locus in the South-East quadrant of Figure 2 which equalizes profits in the delegated and non-cooperative games. That locus must lie below $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ for the following reason. Along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ optimal R&D expenditures are equal in both the delegated and non-cooperative games. However, the laboratory is less efficient than in-house R&D when there are negative indirect externalities. It follows that aggregate benefits in the non-cooperative game exceed those in the delegated game along that locus. As the laboratory does not earn negative profits, $\pi^{\mathrm{d}} < \pi^{\mathrm{n}}$ along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$. Therefore $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ lies above $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$.

[Insert figure 2 about here]

Figure 2 graphs $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ and $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ to compare firms' profits in the three games. As expected, firms' profits are highest in the delegated game when both externalities are positive. However, delegated R&D may yield the lowest profits even if direct externalities are weakly negative and indirect externalities are weakly positive (region below $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ in Figure 2). This occurs because in that region the laboratory earns positive benefits and indirect externalities do not have a high enough impact on aggregate benefits. Hence, positive indirect externalities are necessary but not sufficient for firms to prefer the delegated game to the other two. Note that the firms' profits results have a benchmark flavor, in the sense that the net benefits obtained by a laboratory endowed with some informational advantage, would be bounded from below by the equilibrium benefits obtained here.

4.3 Welfare

Lemma 3 (Cooperative, Non-Cooperative, and Delegated Welfare)

i) There exists a continuous frontier $\delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$ *in the externalities plane such that in all TSPNE* $w^{\mathrm{d}} \gtreqless w^{\mathrm{c}}$ *if and only if* $\delta \gtreqless \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$, *with*

$$\begin{aligned} 0 = \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} &< \delta_{\mathcal{L}=0} \quad \textit{for} \quad \beta < \widetilde{\beta}; \\ 0 = \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} &= \delta_{\mathcal{L}=0} \quad \textit{for} \quad \beta = \widetilde{\beta}; \\ 0 = \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} &> \delta_{\mathcal{L}=0} \quad \textit{for} \quad \beta > \widetilde{\beta}. \end{aligned}$$

ii) There exists a continuous frontier $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$ *in the externalities plane such that in all TSPNE* $w^{\mathrm{d}} \gtreqless w^{\mathrm{n}}$ *if and only if* $\delta \gtreqless \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$, *with*

$$\begin{aligned} 0 < \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} < \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} &\quad \textit{for} \quad \beta < \widetilde{\beta}; \\ \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} = \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} = 0 &\quad \textit{for} \quad \beta = \widetilde{\beta}; \\ 0 > \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} &\quad \textit{for} \quad \beta > \widetilde{\beta}. \end{aligned}$$

The frontier $\delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$ is the direct consequence of Property 2, Proposition 1, and aggregate benefits being increasing in indirect externalities. To understand the intuition for $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$, let direct externalities be negative (i.e., $\beta < \theta/2$). If $\delta = 0$ in that region, both optimal R&D and firms' profits in the delegated game are smaller than in the non-cooperative game by Lemmas 1-(*ii*) and 2-(*ii*) respectively. Therefore, when indirect externalities are negative, $w^{\mathrm{d}} < w^{\mathrm{n}}$ along $\delta = 0$. Second, aggregate benefits in the delegated game must be greater than in the non-cooperative game along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ because the laboratory is more efficient than in-house R&D facilities, and by definition the same amount of R&D is performed in both games. Moreover, w^{d} is increasing in δ (see Lemma D-3 in Appendix D). It follows that for each β in the region bounded by $\delta = 0$ and $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, there exists a value for δ such that welfare in the delegated and non-cooperative games are equal. The existence of $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$ in the South-East quadrant of the externalities plane can be rationalized in the same way.

[**Insert figure 3 about here**]

4.4 Pareto Optimal R&D Organization and Policy Discussion

The juxtaposition of Proposition 2, Lemmas 1, 2 and 3 in the externalities plane allows us to investigate whether one of the three games can Pareto-dominate the other two.

Theorem 1 *The frontiers established in Proposition 2 and Lemmas 1, 2 and 3 are such that:*

$$\begin{aligned}
&\delta_{\mathcal{L}=0} > \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} > \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} > \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} > \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} = \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0 \quad \textit{for } 0 \le \beta < \widetilde{\beta};\\
&\delta_{\mathcal{L}=0} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} = \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} = \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} = \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} = \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0 \quad \textit{for } \beta = \widetilde{\beta};\\
&0 = \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} = \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}} > \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}} > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} > \delta_{\mathcal{L}=0} \quad \textit{for } \widetilde{\beta} < \beta \le 1.
\end{aligned} \tag{16}$$

All frontiers are defined on $[-\gamma, \gamma)$, and the fact they intersect for $\beta = \widetilde{\beta}$ stems from Proposition 3. For (β, δ) such that $0 \le \beta < \widetilde{\beta}$ and $\delta_{\mathcal{L}=0} < \delta < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$, the laboratory earns positive benefits (as opposed to zero profits otherwise). Moreover, in that region, consumer surplus (as inferred from R&D outcomes), and firms' equilibrium profits, are greater in the delegated game than in the cooperative and non-cooperative games. This does not hold elsewhere in the externalities plane, as can be checked from (16).

Corollary 1 (Delegation Dominance) *The delegated* R&D *game Pareto dominates the other two games, and the laboratory earns positive profits, for* $0 \leq \beta < \widetilde{\beta}$ *and* $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} < \delta < \delta_{\mathcal{L}=0}$.

We have therefore established that for certain levels of externalities, consumers, firms, and the laboratory all benefit from the delegation of R&D. Therefore, delegated R&D is a Pareto optimal organizational form. For simple reasons, this cannot occur when direct and indirect externalities are positive. In that case, the delegated game yields highest profits, and consumer surplus, but the laboratory earns no benefits because firms' interests are congruent. For opposite reasons, welfare is minimized under the delegated game if both direct and indirect externalities are sufficiently negative, although in this case a laboratory would earn positive profits. What is crucial for the delegated R&D game to Pareto dominate the other two games, is that indirect externalities must not be too high, so that the firms must still compete for the laboratory's resources, which thus earns positive benefits and participates. But indirect externalities must be high enough to make welfare greater than in the other two games, and let firms obtain more of it than under the two other options.

[**Insert figure 4 about here**]

We can now use these results to examine when the interests of firms and consumers conflict or coincide. This is an important question because firms decide to delegate R&D only if it is profitable for them to do so, and if the laboratory participates. We find that, although no one asks for consumers' consent, the privately profitable decision to delegate R&D is always socially optimal. To see that, remark first that in all three games the consumer surplus increases with R&D because lower costs lead to higher quantities and lower prices (see Appendix D-3). Second, in the externalities plane, for all values of the direct Spillovers parameter, it is more profitable for firms to delegate R&D, than to rely on their own resources or to cooperate, if and only if δ is above $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ (Theorem 1). Now remark that $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ is the "highest" profit frontier and that it is also always above the two frontiers $\delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$ and $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ which allow us to compare the consumer surpluses obtained in all three games equilibria (see Figures 1 and 2). Hence, it is never the case that firms find it most profitable to delegate R&D with consumers being worse off than in either of the two other games (as would be the case if we had, say, $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}} < \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$ for some values of β).

A more striking result is obtained when the laboratory must earn strictly positive profits to participate (i.e., $\mathcal{L} > 0$ is substituted for (10)). This occurs if the laboratory has an outside option where it can earn some arbitrarily small positive net benefits. In that case, firms will delegate R&D only if δ is between $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$ and $\delta_{\mathcal{L}=0}$ when direct externalities are negative (i.e., $0 \leq \beta < \theta/2$). They will never be able to rely on the laboratory's R&D services when direct externalities are positive (i.e. $\theta/2 < \beta \leq 1$). The latter claim arises because the frontier $\delta_{\mathcal{L}=0}$ lies strictly below the two frontiers $\delta_{\pi^{\mathrm{d}}=x^{\mathrm{n}}}$ and $\delta_{\pi^{\mathrm{d}}=x^{\mathrm{c}}}$ when spillovers are high (see Figure 2). Consequently, when the laboratory must make positive benefits to participate, firms will profitably delegate the production of R&D services only when externalities, as represented by points (β, δ), fall in the Pareto dominating region defined in Corollary 1 (see Figure 4). A straightforward policy implication is that, when firms behave as described here, there is no motivation for a regulator to constrain firms' choice to delegate R&D.

5 Conclusion

This paper ranks the outcomes of three R&D games: (i) cooperative in-house R&D, (ii) non-cooperative in-house R&D, and (iii) delegated R&D to an independent profit-maximizing laboratory. In the last game, the introduction of R&D delegation has been kept as simple as consistent with making the comparison possible with the former two standard games of reference. Inter-firm technological spillovers and within-laboratory complementarities are identified as direct and indirect externalities, respectively.

We investigate the impact of these externalities on R&D outcomes, firms' profits, social welfare, and the laboratory's ability to appropriate innovation benefits in a framework with Bertrand competition on the goods market.

Our main results are as follows. First, the laboratory earns positive benefits in the delegated R&D game either if indirect externalities are not too positive when direct externalities are negative, or indirect externalities are sufficiently negative when direct externalities are positive. Second, sufficiently high direct and indirect externalities are necessary and sufficient for optimal R&D, firms' profits, and social welfare to be highest in the case of delegation. Thirdly, a set of points in the externalities plane exists, for which the laboratory participate *and* the delegated R&D game Pareto-dominates the other two games. This holds only when direct externalities are sufficiently negative (in which case firms do compete on the intermediate market for R&D), and indirect externalities positive (i.e., there are economies of scope in the production of R&D), but not too high (otherwise they counteract direct externalities to make the lab only break even). Finally, our findings have a *laissez-faire* flavor, as they do not support any regulatory intervention in the decision by firms to delegate R&D to a profit-seeking provider. This strong statement results from the fact that, in this model, the privately profitable choice to contract with a laboratory also benefits consumers.

These findings are qualitatively similar to those obtained by Vencatachellum and Versaevel (2004) in the same framework where firms compete à la Cournot on the goods market. In that respect, the results are robust to allowing for a change in competition on the final market.

Appendix

A Explicit Solutions of the three R&D Games

We proceed backwards. Section A.1 solves for each firm's price on the final market, which is common to the three games. Then Section A.2 solves for a firm's symmetric R&D in each game.

A.1 Final Market Stage

Each firm chooses its price to maximize (4). This yields two reaction functions, which we use to solve for each firm's subgame Bertrand-Nash equilibrium price as a function $\mathbf{x}$:

$$p_i(\mathbf{x}) = \frac{(2+\theta)\,(c + a(1-\theta)) - (2+\theta\beta)x_i - (2\beta+\theta)x_j}{(2+\theta)(2-\theta)}, \tag{A.1}$$

for $i, j = 1, 2$, $i \neq j$. Substituting (A.1) into (2) we obtain:

$$q_i(p_i(\mathbf{x}), p_j(\mathbf{x})) = \frac{(2+\theta)\,(1-\theta)\,\alpha + \left(2 - \beta\theta - \theta^2\right) x_i + \left(2\beta - \theta - \beta\theta^2\right) x_j}{(2+\theta)\,(2-\theta)\,(1-\theta^2)\,b} \equiv q_i(\mathbf{x}), \tag{A.2}$$

where $\alpha \equiv a - c$, for $i, j = 1, 2$ and $i \neq j$. Using (A.1) and (A.2) into (4) we obtain firm i's concentrated profits as:

$$\pi_i(\mathbf{x}) \;\; = \;\; b\left(1-\theta^2\right)\left[q_i(\mathbf{x})\right]^2, \tag{A.3}$$

for $i = 1, 2$.

A.2 R&D Stage

Define the following terms:

$$\begin{aligned}\Gamma_1 &\equiv b\gamma\,(1+\theta)\,(2-\theta)^2 - 2\,(1-\theta)\,(1+\beta)^2\,, && \text{(A.4)}\\ \Gamma_2 &\equiv b\gamma\left(\theta^2-1\right)(2+\theta)^2\,(2-\theta)^2 + 2\left(2-\beta\theta-\theta^2\right)^2, && \text{(A.5)}\\ \Gamma_3 &\equiv b(\gamma-\delta)(1+\theta)(2-\theta)^2 - 2(1-\theta)(1+\beta)^2. && \text{(A.6)}\end{aligned}$$

In the following we assume Γ_1, Γ_2 and Γ_3 are positive. This ensures the objective functions in the cooperative, non-cooperative, and delegated games, respectively, are concave. We also define:

$$\Gamma_4 = b\gamma\,(1+\theta)\,(2+\theta)\,(2-\theta)^2 - 2\,(1+\beta)\left(2-\beta\theta-\theta^2\right), \tag{A.7}$$

and assume it is positive. This will simplify the notation in what follows.

In the non-cooperative case, Henriques (1990) establishes that reaction functions in the R&D space cross "correctly" when $|\partial x_i/\partial x_j|$ is less than 1. In our case this condition becomes:

$$-2\big|\left(\theta^2+\beta\theta-2\right)\left(\beta\theta^2+\theta-2\beta\right)\big|/\Gamma_2 < 1. \tag{A.8}$$

Note that(A.8) holds for all values of β when $b=1$, $\theta=\frac{3}{4}$, and $\gamma=2$, as is the case in Figures 1 to 4.

A.2.1 Cooperative R&D

Let firm i's net profit as a function of $\mathbf{x}$ be denoted by:

$$\pi_i \equiv \pi_i(\mathbf{p},\mathbf{x}) - r(x_i), \tag{A.9}$$

where $\mathbf{p} \equiv (p_1, p_2)$. We maximize $\pi_1(\mathbf{x}) + \pi_2(\mathbf{x})$ with respect to $\mathbf{x}$ to obtain the symmetric cooperative R&D outcome (x^{c}) and individual firm profits (π^{c}), respectively, as:

$$x^{\mathrm{c}} = 2\alpha\,(1-\theta)\,(1+\beta)/\Gamma_1, \tag{A.10}$$

$$\pi^{\mathrm{c}} = \gamma\alpha^2(1-\theta)/\Gamma_1. \tag{A.11}$$

Both (A.10) and (A.11) are non-negative because $\theta \in [0,1)$, $\beta \in [0,1]$, and $\Gamma_1 > 0$ by assumption.

A.2.2 Non-Cooperative R&D

Each firm chooses its R&D independently to maximize (A.9). This yields two reaction functions, which we use to solve for a symmetric non-cooperative R&D outcome and can then substitute into (A.9) to obtain the symmetric individual firm profits. Hence, we have:

$$x^{\mathrm{n}} = 2\alpha\left(2-\theta^2-\beta\theta\right)/\Gamma_4, \tag{A.12}$$

$$\pi^{\mathrm{n}} = \alpha^2\gamma\Gamma_2/\Gamma_4^2. \tag{A.13}$$

Both (A.12) and (A.13) are positive because $\Gamma_2 > 0$ and $\Gamma_4 > 0$ by assumption, while $\theta \in [0,1)$ and $\beta \in [0,1]$.

A.2.3 Delegated R&D

Maximizing (15) with respect to $\mathbf{x}$ gives each firm's symmetric delegated R&D outcome:

$$x^{\mathrm{d}} = 2\alpha\,(1-\theta)\,(1+\beta)/\Gamma_3. \tag{A.14}$$

Substituting (A.14) into (15), and using the laboratory's explicit cost function (14), gives:

$$\Lambda = 2\alpha^2 \left(1 - \theta\right)(\gamma - \delta)/\Gamma_3. \tag{A.15}$$

Both x^{d} and Λ are positive because $\Gamma_3 > 0$ by assumption, and $\delta \in [-\gamma, \gamma)$.

B Proof of Properties 1 and 2

B.1 Proof of Property 1

We use (A.2) and (A.3) to obtain:

$$\frac{\mathrm{d}\pi_i(\mathbf{x})}{\mathrm{d}x_j} = \frac{2\left(2 - \theta^2\right)(\beta - \widetilde{\beta})q_i(\mathbf{x})}{(4 - \theta^2)}, \tag{B.1}$$

for $i, j = 1, 2$, $i \neq j$. As $\theta \in [0, 1]$, and quantities produced are non-negative, (B.1) is of the same sign as $(\beta - \widetilde{\beta})$ for a positive output, or equals 0 otherwise. ■

B.2 Proof of Property 2

The proof follows directly from differentiating the laboratory's cost function (14) with respect to x_i and x_j. ■

C Proof of Propositions

C.1 Proof of Proposition 1

The proof of Proposition 1 is a simple adaptation in the notation from Bernheim and Whinston (1986, first part of Theorem 2 on page 14, and proof on pages 24-25). It is available upon request from the authors.

C.2 Proof of Proposition 2

Let $N = \{1, 2\}$, and $2^N = \{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$. We build on Laussel and Le Breton (2001) – henceforth LLB - by associating to the common agency game, as defined in section 2.3, a function $\pi : 2^N \rightarrow \mathrm{R}$, such that:

$$\pi(M) = \max_{\mathbf{x}} \sum_{i \in M} \pi_i(\mathbf{x}) - s(\mathbf{x}), \tag{C.1}$$

where $s(\mathbf{x})$ is given by (14) and $\pi_i(\mathbf{x})$ by (A.3). This function gives the highest joint-benefits of the laboratory and any subset M of firms in N, with $\pi(\emptyset) = \pi_1(0, 0) + \pi_2(0, 0)$, that is the sum of concentrated profits with no R&D (a normalization). Then the proof consists in investigating additive properties of π on 2^N in order to exploit a series of theorems that characterize the equilibrium outcomes of the delegated R&D game.

- From LLB's Theorem 3.1 (p. 102), if $\pi(M)$ is strictly subadditive, that is:

$$\pi(\{1, 2\}) < \pi(\{1\}) + \pi(\{2\}), \tag{C.2}$$

then the laboratory earns positive benefits in all equilibria, that is $\mathcal{L} > 0$.

- From LLB's Theorem 3.3 (p. 104), if $\pi(M)$ is strictly subadditive, then $\#N = 2$ implies that firms' symmetric profits in the delegated R&D game are:

$$\pi^{\mathrm{d}} = \Lambda - \pi(\{i\}), \tag{C.3}$$

$i = 1, 2$.

- From LLB's Theorem 3.2 (p. 103), if $\pi(M)$ is superadditive, that is:

$$\pi(\{1,2\}) \geq \pi(\{1\}) + \pi(\{2\}), \tag{C.4}$$

then $\mathcal{L} = 0$. In that case, symmetry in firms' gross profit functions, and the fact that the laboratory maximizes aggregate benefits $\Lambda(\mathbf{x})$ in $(\mathbf{x})$ (Proposition 1), imply that a firm's profits in the delegated game are:

$$\pi^{\mathrm{d}} = \Lambda/2. \tag{C.5}$$

The remainder of the proof identifies values of δ which are such that $\pi(M)$ is either strictly subadditive or superadditive. To that effect we solve for values of δ such that (C.4) holds with equality, to obtain a frontier which we denote by $\delta_{\mathcal{L}=0}$. However, the free maximization of $\pi(M)$, for $M = \{i\}$, $i = 1, 2$, may yield negative maximands. Therefore, we consider in turn the free-maximum and constrained-maximum versions of (C.1), denoted by $\check{\pi}(\{i\})$ and $\hat{\pi}(\{i\})$, respectively. We thus obtain two frontiers $\check{\delta}_{\mathcal{L}=0}$ (free-maximum) and $\hat{\delta}_{\mathcal{L}=0}$ (constrained-maximum) each of which verify (C.4) with equality. We then calculate the values of δ for which the free maximand $\check{x}_j$ is equal to the constrained variable $\hat{x}_j \equiv 0$, and denote it by $\delta_{\check{x}_j=\hat{x}_j=0}$. Finally, we compare $\hat{\delta}_{\mathcal{L}=0}$ and $\check{\delta}_{\mathcal{L}=0}$ with $\delta_{\check{x}_j=\hat{x}_j=0}$ to verify for which parameter values (i) $\check{\delta}_{\mathcal{L}=0}$ verifies the positive maximand constraint, and (ii) $\hat{\delta}_{\mathcal{L}=0}$ verifies the non-positive maximand constraint. This allows us to derive the frontier $\delta_{\mathcal{L}=0}$ by "pasting" those two functions.

C.2.1 Free and Constrained Solutions

Let $\Phi \equiv \left(2-\beta\theta-\theta^2\right)$, $\Psi \equiv \left(2\beta-\theta-\beta\theta^2\right)$, and define the following three terms:

$$\Gamma_5 = b\gamma\left(1-\theta^2\right)\left(4-\theta^2\right)^2 - 2\Phi^2,$$

$$\Gamma_6 = b\gamma\left(1-\theta^2\right)\left(4-\theta^2\right)^2 - 2\Psi^2,$$

$$\Gamma_7 = b\left(\gamma^2-\delta^2\right)\left(1-\theta^2\right)\left(4-\theta^2\right)^2 - 2\left(\Phi^2+\Psi^2\right)\gamma - 4\Phi\Psi\delta.$$

We assume Γ_5, Γ_6 and Γ_7 are positive to ensure the objective function of the free-maximum problem is concave, while $\Gamma_5 > 0$ is sufficient for the objective function of the constrained-maximum problem to be concave.

- Firstly, we solve the free-maximum version of (C.1), with $M = \{i\}$, $i = 1, 2$. In that case:

$$\breve{\pi}\left(\{i\}\right) = \max_{\mathbf{x}}\left(\pi_i(\mathbf{x}) - s(\mathbf{x})\right), \tag{C.6}$$

where $s(\mathbf{x})$ is given by (14) and $\pi_i(\mathbf{x})$ by (A.3). Maximizing the right-hand side of (C.6) gives the following unconstrained R&D solutions:

$$\breve{x}_i = 2\alpha\left(1-\theta\right)\left(2+\theta\right)\left(\gamma\Phi+\delta\Psi\right)/\Gamma_7, \tag{C.7}$$

$$\breve{x}_j = 2\alpha\left(1-\theta\right)\left(2+\theta\right)\left(\gamma\Psi+\delta\Phi\right)/\Gamma_7. \tag{C.8}$$

Making use of (C.7) and (C.8) into (C.6) we obtain, for $i = 1, 2$:

$$\breve{\pi}\left(\{i\}\right) = \alpha^2\left(1-\theta\right)^2\left(2+\theta\right)^2\left(\gamma^2-\delta^2\right)/\Gamma_7. \tag{C.9}$$

- Secondly, we solve the constrained-maximum versions of (C.1), with $M = \{i\}$, $i = 1, 2$, and given by:

$$\hat{\pi}\left(\{i\}\right) = \max_{x_i}\left(\pi_i(x_i, 0) - r(x_i)\right), \tag{C.10}$$

where $r(x_i)$ is given by (5), and $\pi_i(x_i, 0)$ is obtained by setting x_j equal to 0 in (A.3). Maximizing the right-hand side of (C.10) gives the constrained R&D solution:

$$\hat{x}_i = 2\alpha\left(1-\theta\right)\left(2+\theta\right)\Phi/\Gamma_5, \qquad \text{for} \quad i = 1, 2. \tag{C.11}$$

Substituting (C.11) into the right-hand side of (C.10) gives:

$$\hat{\pi}\left(\{i\}\right) = \gamma\alpha^2\left(1-\theta\right)^2\left(2+\theta\right)^2/\Gamma_5, \qquad \text{for} \quad i = 1, 2. \tag{C.12}$$

- We can now derive the frontier $\delta_{\breve{x}_j = \hat{x}_j = 0}$, i.e. all values of δ which are such that $\breve{x}_j = \hat{x}_j \equiv 0$. This yields:

$$\breve{x}_j \quad \substack{>\\=\\<} \quad \hat{x}_j \equiv 0 \quad \text{if and only if} \quad \delta \quad \substack{>\\=\\<} \quad -\gamma\frac{\Psi}{\Phi}. \tag{C.13}$$

C.2.2 The Laboratory's Zero-Benefits Free-maximum Frontier

We evaluate (C.4), assuming it holds with equality with free-maximum profits:

$$\pi(\{1, 2\}) - \breve{\pi}(\{1\}) - \breve{\pi}(\{2\}) = 0, \tag{C.14}$$

where $\breve{\pi}(\{1\})$ and $\breve{\pi}(\{2\})$ are given by (C.9). The solution to (C.14) in δ give five non-admissible roots ($a = c$, $\beta = \frac{2-\theta^2}{\theta}$ and $\delta = 0$, $\gamma = \delta$, and $\theta = 1$) and one which is admissible:

$$\delta = -\frac{\Psi\Phi}{\Psi^2 + \Phi^2}. \tag{C.15}$$

To check that (C.15) is compatible with free maximands, recall from (C.13) that (C.15) is defined only if $\check{\delta}_{\mathcal{L}=0} \geq \delta_{\check{x}_j=\hat{x}_j=0}$. We form the difference:

$$\delta_{\check{x}_j=\hat{x}_j=0} - \check{\delta}_{\mathcal{L}=0}, \tag{C.16}$$

and look for parameters values for which it is non-positive. Equating (C.16) to 0, gives six non admissible roots ($\beta = -1$, $\gamma = 0$, $\theta = -2$, $\theta = -1$, $\theta = 1$, and $\theta = 2$), and two which are admissible: $\beta = \widetilde{\beta}$ and $\beta = 1$. Hence, (C.16) changes sign once in the domain of β. Evaluating (C.16) at some parameter values, say $(\beta, \gamma, \theta) = (1/2, 1, 1/2)$, we obtain $\check{\delta}_{\mathcal{L}=0} < \delta_{\check{x}_j=\hat{x}_j=0}$. It follows that $\check{\delta}_{\mathcal{L}=0}$ is defined only for $0 \leq \beta \leq \widetilde{\beta}$.

Finally, using (C.15), note that $\check{\delta}_{\mathcal{L}=0}$ includes the pivotal point.

C.2.3 The Laboratory's Zero-Benefits Constrained-maximum Frontier

Similar to C.2.2 we solve:

$$\pi(\{1,2\}) - \hat{\pi}(\{1\}) - \hat{\pi}(\{2\}) = 0, \tag{C.17}$$

where the constrained profits $\hat{\pi}(\{1\})$ and $\hat{\pi}(\{2\})$ are given by ((C.11). Equation (C.17) has two non admissible roots ($a = c$, and $\theta = 1$), and one which is admissible:

$$\delta = -\gamma \frac{2\Phi\Psi + \Psi^2}{\Phi^2} \equiv \hat{\delta}_{\mathcal{L}=0}. \tag{C.18}$$

To check that the latter expression is compatible with a constrained maximand, recall from (C.13) that (C.18) is defined only if $\hat{\delta}_{\mathcal{L}=0} \leq \delta_{\check{x}_j=\hat{x}_j=0}$. Then form the difference:

$$\delta_{\check{x}_j=\hat{x}_j=0} - \hat{\delta}_{\mathcal{L}=0}, \tag{C.19}$$

and look for the parameter values for which it is non-negative. Equating (C.19) to 0, we obtain four non admissible roots ($\beta = -1$, $\gamma = 0$, $\theta = -2$, and $\theta = 1$), and one which is admissible: $\beta = \widetilde{\beta}$. Hence, (C.19) changes sign only once over the domain of β. Evaluating (C.19) at some parameter values, say $(\beta, \gamma, \theta) = (0, 1, 1/2)$, gives $\hat{\delta}_{\mathcal{L}=0} > \delta_{\check{x}_j=\hat{x}_j=0}$. It follows that (C.18) is defined only for $\widetilde{\beta} \leq \beta \leq 1$.

Finally, using (C.18), note that, as for $\check{\delta}_{\mathcal{L}=0}$, $\hat{\delta}_{\mathcal{L}=0}$ also includes the pivotal point.

C.2.4 The Laboratory's Zero-Benefits Frontier

C.2.3 and C.2.4 together means that $\pi(\{1,2\}) = \hat{\pi}(\{1\}) + \hat{\pi}(\{2\})$ if and only if:

$$\delta = \delta_{\mathcal{L}=0} = \begin{cases} \check{\delta}_{\mathcal{L}=0} > \delta_{\check{x}_j=\hat{x}_j=0} > 0 & \text{for} \quad 0 \leq \beta \leq \widetilde{\beta}, \\ \hat{\delta}_{\mathcal{L}=0} < \delta_{\check{x}_j=\hat{x}_j=0} < 0 & \text{for} \quad \widetilde{\beta} \leq \beta \leq 1, \end{cases} \tag{C.20}$$

where $\check{\delta}_{\mathcal{L}=0}$ and $\hat{\delta}_{\mathcal{L}=0}$ are given by (C.15) and (C.18). Note that $\beta = \widetilde{\beta}$ implies $\delta_{\mathcal{L}=0} = \check{\delta}_{\mathcal{L}=0} = \hat{\delta}_{\mathcal{L}=0} = 0$.

We now prove that $\delta_{\mathcal{L}=0}$ is decreasing in β, by considering $\check{\delta}_{\mathcal{L}=0}$ and $\hat{\delta}_{\mathcal{L}=0}$ in turn.

($\check{\delta}_{\mathcal{L}=0}$) Differentiating (C.15) with respect to β and equating to 0 yields seven roots ($\beta = -1$, $\beta = 1$, $\gamma = 0$, $\theta = -2$, $\theta = -1$, $\theta = 1$, and $\theta = 2$), none of which is is admissible. It follows that $\check{\delta}_{\mathcal{L}=0}$ is strictly monotone over the domain of β on which it is defined, that is $[0, \widetilde{\beta}]$. To complete the proof, let for instance $\beta = \widetilde{\beta}$, and check that $\mathrm{d}\check{\delta}_{\mathcal{L}=0}/\mathrm{d}\beta < 0$, as required.

($\hat{\delta}_{\mathcal{L}=0}$) Differentiating (C.18) with respect to β and equating to 0 yields six roots ($\beta = -1$, $\gamma = 0$, $\theta = -2$, $\theta = -1$, $\theta = 1$, and $\theta = 2$), none of which is admissible. It follows that $\hat{\delta}_{\mathcal{L}=0}$ is strictly monotone over the domain of β on which it is defined, that is $[\widetilde{\beta}, 1]$. To complete the proof, let for instance $\beta = 1$, and check that $\mathrm{d}\hat{\delta}_{\mathcal{L}=0}/\mathrm{d}\beta < 0$. ■

C.3 Proof of Proposition 3

Firstly, if $\beta = \widetilde{\beta}$, for all δ, concentrated profits $\pi_i(\mathbf{x})$ depend only on each firm i's own R&D variable x_i (Property 1), in which case the cooperative and non-cooperative games coincide. Secondly, if $\delta = 0$, for all β, we have $r(x_1) + r(x_2) = s(\mathbf{x})$ (Property 2), and solving the cooperative game is equivalent to solving the delegated game (Proposition 1). Thirdly, if $\beta = \widetilde{\beta}$ and $\delta = 0$, the laboratory earns no benefits (Proposition 2). By considering all three cases together, we conclude that the cooperative, non-cooperative, and delegated R&D games yield identical outcomes at the non-externalities point $(\widetilde{\beta}, 0)$. $\blacksquare$

D Proof of Lemmas

We first establish how indirect externalities impact optimal outcomes in the delegated R&D game. This will be useful for the proofs of Lemmas 1–3, which follow.

Lemma D-1 (R&D) $\mathrm{d}x^{\mathrm{d}}/\mathrm{d}\delta > 0$.

Proof. Differentiating (A.14) with respect to δ gives:

$$\frac{\mathrm{d}x^{\mathrm{d}}}{\mathrm{d}\delta} = 2\alpha b\,(1+\beta)\left(1-\theta^2\right)(2-\theta)^2/\Gamma_3^2. \tag{D.1}$$

The result follows from that noting $\Gamma_3 > 0$ by assumption and $\theta \in [0, 1)$. $\blacksquare$

Lemma D-2 (Profits) $\mathrm{d}\pi^{\mathrm{d}}/\mathrm{d}\delta > 0$.

Proof We consider $\delta > \delta_{\mathcal{L}=0}$ and $\delta \leq \delta_{\mathcal{L}=0}$ in turn.

$(\delta \geq \delta_{\mathcal{L}=0})$ By Propositions 1 and 2:

$$\pi^{\mathrm{d}} = \Lambda/2. \tag{D.2}$$

Recalling that the laboratory's cost are given by (14), aggregate benefits by (15), and using the envelope theorem, we differentiate (D.2) with respect to δ to obtain:

$$\frac{\mathrm{d}\pi^{\mathrm{d}}}{\mathrm{d}\delta} = \frac{1}{2}\frac{\partial \Lambda}{\partial \delta} = \frac{1}{2}(x^{\mathrm{d}})^2, \tag{D.3}$$

which is unambiguously positive because $x^{\mathrm{d}} > 0$ for $\theta \in [0, 1)$.

$(\delta \leq \delta_{\mathcal{L}=0})$ Suppose π^{d} is non increasing in δ and look for a contradiction. The monotonicity of π^{d} implies that:

$$\pi^{\mathrm{d}}\Big|_{\delta<\delta_{\mathcal{L}=0}} \geq \pi^{\mathrm{d}}\Big|_{\delta=\delta_{\mathcal{L}=0}}. \tag{D.4}$$

Recall from Proposition 1 that the laboratory maximizes aggregate benefits, and from Proposition 2 that it breaks even if $\delta = \delta_{\mathcal{L}=0}$. Consequently:

$$\pi^{\mathrm{d}}\Big|_{\delta=\delta_{\mathcal{L}=0}} = \frac{1}{2}\,\Lambda|_{\delta=\delta_{\mathcal{L}=0}}. \tag{D.5}$$

Moreover, recalling that the laboratory's cost are given by (14) and aggregate benefits by (15), by using the envelope theorem we obtain $\mathrm{d}\Lambda/\mathrm{d}\delta = (x^{\mathrm{d}})^2$, which is unambiguously positive given that $x^{\mathrm{d}} > 0$. It follows that:

$$\frac{1}{2}\,\Lambda|_{\delta=\delta_{\mathcal{L}=0}} > \frac{1}{2}\,\Lambda|_{\delta<\delta_{\mathcal{L}=0}}. \tag{D.6}$$

To conclude, taking (D.4), (D.5), and (D.6) together leads to:

$$\pi^{\mathrm{d}}\Big|_{\delta<\delta_{\mathcal{L}=0}} > \frac{1}{2}\,\Lambda|_{\delta<\delta_{\mathcal{L}=0}}, \tag{D.7}$$

by transitivity. Inequality (D.7) contradicts $\pi^{\mathrm{d}} \leq \Lambda/2$ for all $\delta \leq \delta_{\mathcal{L}=0}$ as established by Propositions 1 and 2. Hence $\mathrm{d}\pi^{\mathrm{d}}/\mathrm{d}\delta > 0$. ■

Lemma D-3 (Welfare) $\mathrm{d}w^{\mathrm{d}}/\mathrm{d}\delta > 0$.

Proof. Welfare is the sum of firms' profits and consumers' surplus. It was established in Lemma D-2 that π^{d} are increasing in δ. Here we turn to consumers' surplus by investigating how $q_i(\mathbf{x}^{\mathrm{d}})$ and $p_i(\mathbf{x}^{\mathrm{d}})$ vary with δ.

($q_i(\mathbf{x}^{\mathrm{d}})$) Differentiating the demand function (2) evaluated at $\mathbf{x}^{\mathrm{d}} = (x^{\mathrm{d}}, x^{\mathrm{d}})$, with respect to δ, we obtain:

$$\frac{\mathrm{d}q_i(\mathbf{x}^{\mathrm{d}})}{\mathrm{d}\delta} = (2-\theta)(1+\beta)\,x^{\mathrm{d}}/\Gamma_3. \tag{D.8}$$

As $\Gamma_3 > 0$ by assumption, $\theta \in [0,1)$, and $\beta \in [0,1]$ it follows that (D.8) is also positive.

($p_i(\mathbf{x}^{\mathrm{d}})$) Differentiating the Bertrand-Nash symmetric equilibrium price (A.1) evaluated at $\mathbf{x}^{\mathrm{d}} = (x^{\mathrm{d}}, x^{\mathrm{d}})$, with respect to δ, we obtain:

$$\frac{\mathrm{d}p_i(\mathbf{x}^{\mathrm{d}})}{\mathrm{d}\delta} = -b\,(2-\theta)(1+\theta)(1+\beta)\,x^{\mathrm{d}}/\Gamma_3. \tag{D.9}$$

As $\Gamma_3 > 0$, $\theta \in [0,1)$, $\beta \in [0,1]$ and x^{d} is non negative, it follows that (D.9) is negative.

Taking (D.8) and (D.9) together means that the consumer surplus is increasing in δ. The fact that firms' profits are also increasing in δ (Lemma D-2) completes the proof. ■

D.1 Proof of Lemma 1

D.1.1 Equal Delegated and Cooperative R&D Frontier $\delta_{x^{\mathrm{d}}=x^{\mathrm{c}}}$

As x^{d} is monotone increasing in δ (Lemma D-1) and x^{c} is invariant with δ, it follows that if there exists a value of δ for which $x^{\mathrm{d}} = x^{\mathrm{c}}$, it is unique. Moreover, for $\delta = 0$, (i) the costs of R&D are the same for the laboratory in the delegated R&D and for both firms in the cooperative game (see Property 2), and (ii) solving the delegated game for x^{d} is equivalent to solving the cooperative game for x^{c} (because of Proposition 1). Hence, $x^{\mathrm{d}} = x^{\mathrm{c}}$ for $\delta = 0$. Using that result, Lemma D-1, and Proposition 2 gives Lemma 1-(i). ■

D.1.2 Equal Delegated and Non-Cooperative R&D Frontier $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$

Using (A.12) and (A.14) we define:

$$\Delta(\delta) \equiv x^{\mathrm{d}} - x^{\mathrm{n}}. \tag{D.10}$$

$\beta = \widetilde{\beta}$ $\Delta(0) = 0$ because of Proposition 3.

$\beta < \widetilde{\beta}$ Claim A: $\Delta(\delta) < 0$ for $\delta = 0$. Section D.1.1 establishes that $x^{\mathrm{d}} = x^{\mathrm{c}}$ for $\delta = 0$. Next, we know from Hinloopen (2000) that $x^{\mathrm{c}} < x^{\mathrm{n}}$ for $\beta \in [0, \widetilde{\beta})$ and all values of δ. Claim A follows by transitivity.

Claim B: $\Delta(\delta) > 0$ along $\delta_{\mathcal{L}=0}$. Recall from (C.20) that $\delta_{\mathcal{L}=0} = \check{\delta}_{\mathcal{L}=0} > 0$ for $\beta < \widetilde{\beta}$. Then evaluating (D.10) at $\delta = \check{\delta}_{\mathcal{L}=0}$, and equating to 0, gives eight roots ($a = c$, $b = 0$, $\beta = 1$, $\beta = \widetilde{\beta}$, $\gamma = 0$, $\theta = -2$, $\theta = -1$, and $\theta = 2$), none of which is admissible. Therefore, $\Delta(\check{\delta}_{\mathcal{L}=0})$ does not change sign over this range of β. It is straightforward to check that claim B holds by computing $\Delta(\check{\delta}_{\mathcal{L}=0})$ at, say, $\beta = 0$ and any admissible values for the other parameters, and obtaining a positive value.

$\beta > \widetilde{\beta}$ Claim C: $\Delta(\delta) > 0$ for $\delta = 0$. Recall that $x^{\mathrm{d}} = x^{\mathrm{c}}$ for $\delta = 0$, as established in Section D.1.1. Then note that $x^{\mathrm{c}} > x^{\mathrm{n}}$ for $\beta \in (\widetilde{\beta}, 1]$ from Hinloopen (2000), all δ. Claim C follows by transitivity.

Claim D: $\Delta(\delta) < 0$ along $\delta_{\mathcal{L}=0}$. Recall from (C.20) that $\delta_{\mathcal{L}=0} = \hat{\delta}_{\mathcal{L}=0} < 0$ for $\beta > \widetilde{\beta}$. Then evaluating (C.20) at $\delta = \hat{\delta}_{\mathcal{L}=0}$, and equating to 0, gives eight roots ($a = c$, $b = 0$, $\beta = \widetilde{\beta}$, $\beta = \frac{2-\theta^2}{\theta}$, $\gamma = 0$, $\theta = -2$, $\theta = -1$, and $\theta = 2$), none of which is admissible. Therefore, $\Delta(\hat{\delta}_{\mathcal{L}=0})$ does not change sign over the relevant range of β. It is straightforward to check that $\Delta(\hat{\delta}_{\mathcal{L}=0})$ is negative by evaluating it at, say, $\beta = 1$ and any admissible values for the other parameters. Hence claim D is true.

Recall how $\delta_{\mathcal{L}=0}$ is constructed in (C.20), use claims A to D, Lemma D-1, and that x^{n} does not vary with δ, to obtain Lemma 1-(*ii*). ■

D.2 Proof of Lemma 2

D.2.1 Equal Delegated and Cooperative Profits Frontier $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$

$(\beta = \widetilde{\beta})$

We know from Proposition 3 that $\pi^{\mathrm{d}} = \pi^{\mathrm{c}}$ for $\delta = 0$ (the no-externalities case).

Now we consider $\beta < \widetilde{\beta}$, for which $\delta_{\mathcal{L}=0} > 0$, and $\beta > \widetilde{\beta}$, for which $\delta_{\mathcal{L}=0} < 0$.

$(\beta < \widetilde{\beta})$

Claim A: $\pi^{\mathrm{d}} < \pi^{\mathrm{c}}$ for $\delta = 0$.

On the one hand:

$$\pi^{\mathrm{d}} < \frac{1}{2}\,\Lambda|_{\delta=0}\,, \tag{D.11}$$

because the laboratory appropriates a share of maximized aggregate benefits as $\mathcal{L} > 0$ for $\delta = 0 < \delta_{\mathcal{L}=0}$, as can be inferred from Propositions 1 and 2. On the other hand:

$$\pi^{\mathrm{c}} = \frac{1}{2} \left. \Lambda \right|_{\delta=0} , \tag{D.12}$$

from the specification of the cooperative R&D game. Putting (D.11) and (D.12) together gives $\pi^{\mathrm{d}} < \pi^{\mathrm{c}}$ for $\delta = 0$.

Claim B: $\pi^{\mathrm{d}} > \pi^{\mathrm{c}}$ for $\delta = \delta_{\mathcal{L}=0}$.

On the one hand:

$$\pi^{\mathrm{d}} = \frac{1}{2} \left. \Lambda \right|_{\delta=\delta_{\mathcal{L}=0}} , \tag{D.13}$$

from Propositions 1 and 2 because firms earn all maximized aggregate benefits as $\mathcal{L} = 0$ for $\delta = \delta_{\mathcal{L}=0}$. On the other hand:

$$\pi^{\mathrm{c}} < \frac{1}{2} \left. \Lambda \right|_{\delta=\delta_{\mathcal{L}=0}} , \tag{D.14}$$

because of (D.12) and $\left. \Lambda \right|_{\delta=0} < \left. \Lambda \right|_{\delta=\delta_{\mathcal{L}=0}}$ as a result of (D.3). Putting (D.13) and (D.14) together means claim B holds.

$(\beta > \widetilde{\beta})$

Claim C: $\pi^{\mathrm{d}} = \pi^{\mathrm{c}}$ for $\delta = 0$.

On the one hand:

$$\pi^{\mathrm{d}} = \frac{1}{2} \left. \Lambda \right|_{\delta=0} , \tag{D.15}$$

from Propositions 1 and 2 (as $\mathcal{L} = 0$ for $\delta = 0 > \delta_{\mathcal{L}=0}$). On the other hand:

$$\pi^{\mathrm{c}} = \frac{1}{2} \left. \Lambda \right|_{\delta=0} , \tag{D.16}$$

from the specification of the cooperative R&D game. Claim C follows from (D.15) and (D.16).

Claim D: $\delta_{\mathcal{L}=0} < 0$ for $\beta > \widetilde{\beta}$.

Claim D follows directly from Proposition 2.

Claims A to D, and Lemma D-2 combine to give Lemma 2-(i). ■

D.2.2 Equal Delegated and Non-Cooperative Profits Frontier $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$

Using (A.13) and, (C.3) or (C.5), we define:

$$\tilde{\Delta}(\delta) \equiv \pi^{\mathrm{d}} - \pi^{\mathrm{n}}.$$

($\beta = \widetilde{\beta}$) We know from Proposition 3 that $\tilde{\Delta}(0) = 0$.

Now we consider $\beta < \widetilde{\beta}$, for which $\delta_{\mathcal{L}=0} > 0$, and $\beta > \widetilde{\beta}$, for which $\delta_{\mathcal{L}=0} < 0$. In both cases, we make use of $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ as defined in Lemma 1. Note that $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ is identical to $\delta_{\check{x}_j=\hat{x}_j=0}$, for all β. There are six roots to (D.10) equal 0. Five are not admissible ($a = c$, $b = 0$, $\theta = -1$, $\theta = 2$, and $(\beta, \theta) = (1, 1)$), and the only admissible one is $\delta = -\gamma\Psi/\Phi$. The admissible root coincides with the frontier given in (C.13).

($\beta < \widetilde{\beta}$) Claim A: $\tilde{\Delta}(\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}) < 0$.

As $\delta = \delta_{\check{x}_j=\hat{x}_j=0} = \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, this implies that $\pi^{\mathrm{d}} = \Lambda - \pi\left(\{i\}\right)$ from Proposition 2, with $\pi\left(\{i\}\right) = \hat{\pi}\left(\{i\}\right) = \check{\pi}\left(\{i\}\right)$ because of the definition of $\delta_{\check{x}_j=\hat{x}_j=0}$.[11] With this algebraic form for π^{d}, we equate $\tilde{\Delta}(\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}})$ to 0 and solve for the roots. We obtain four roots ($a = c$, $\beta = \widetilde{\beta}$, $\beta = \frac{2-\theta^2}{\theta}$, $\gamma = 0$), none is admissible. Therefore $\tilde{\Delta}(\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}})$ does not change sign. Then by evaluating $\tilde{\Delta}(\delta)$ at, say, $\beta = 0$ and any admissible values for the other parameters, allows us to conclude that claim A is true.

Claim B: $\tilde{\Delta}(\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}) > 0$.

By definition $\pi^{\mathrm{d}} = \pi^{\mathrm{c}}$ along $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$, while $\pi^{\mathrm{c}} > \pi^{\mathrm{n}}$ for $\beta < \widetilde{\beta}$ and any δ (Hinloopen 2000). Therefore, by transitivity, claim B is true.

$(\beta > \widetilde{\beta})$ Claim C: $\tilde{\Delta}(\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}) < 0$.

Recall from (3) that the unit costs of production are equal under delegated and non-cooperative R&D along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$. Thus, from (2) and (A.1), $q_i(\mathbf{x}^{\mathrm{d}}) = q_i(\mathbf{x}^{\mathrm{n}})$ and $p_i(\mathbf{x}^{\mathrm{d}}) = p_i(\mathbf{x}^{\mathrm{n}})$, for $i = 1, 2$, along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$. It follows that gross concentrated profits (i.e., before R&D costs) are also equal, that is:

$$\pi_i(\mathbf{x}^{\mathrm{d}}) = \pi_i(\mathbf{x}^{\mathrm{n}}), \tag{D.17}$$

$i = 1, 2$. Moreover, we know from Lemma 1-(*ii*) that $\delta_{\mathcal{L}=0} < \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} < 0$. The first inequality sign means that the laboratory exactly breaks even along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ because of Proposition 2. This implies that firms' symmetric transfer payments exactly cover the laboratory's costs, that is $t_1^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}}) + t_2^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}}) = s(\mathbf{x}^{\mathrm{d}})$. The second inequality means indirect externalities are negative along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, because of Property 2. This implies that the laboratory's R&D costs are strictly greater than the firms' total R&D costs, that is $s(\mathbf{x}^{\mathrm{d}}) > r(x_1^{\mathrm{n}}) + r(x_2^{\mathrm{n}})$, with $x_1^{\mathrm{n}} = x_2^{\mathrm{n}} = x^{\mathrm{n}}$. It follows that $t_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}}) > r(x_i^{\mathrm{n}})$ along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$. It suffices to use (D.17) to obtain:

$$\pi_i(\mathbf{x}^{\mathrm{d}}) - t_i^{\mathrm{d}}(\mathbf{x}^{\mathrm{d}}) < \pi_i(\mathbf{x}^{\mathrm{n}}) - r(x_i^{\mathrm{n}}), \tag{D.18}$$

$i = 1, 2$. Inequality (D.18) says that claim C, which refers to net profits, is true.

Claim D: $\widetilde{\Delta}(0) > 0$.

In the absence of indirect externalities, we have $s(\mathbf{x}) = r(x_1)+r(x_2)$. In that case, from Proposition 1, the optimal R&D is the same in the delegated and cooperative games. Then, from Proposition 2, because $\delta = 0 > \delta_{\mathcal{L}=0}$ implies that the laboratory exactly breaks even, we have $\pi^{\mathrm{d}} = \pi^{\mathrm{c}}$. As π^{c} is always greater than π^{n}, from Hinloopen (2000), claim D follows by transitivity.

Given claims A to D, Lemma D-2, and that π^{n} is invariant with δ, we obtain Lemma 2-(*ii*). ■

D.3 Proof of Lemma 3

D.3.1 Equal Delegated and Cooperative Welfare Frontier $\delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$

As w^{d} is monotone increasing in δ (Lemma D-3), and w^{c} does not vary with δ, if there exists a value of δ such that $w^{\mathrm{d}} = w^{\mathrm{c}}$, it is unique. Moreover, for $\delta = 0$, we know (i) the laboratory's costs in the delegated game are equal to both firms' total R&D costs in the cooperative game (Property 2), and (ii) solving the delegated game for x^{d} is equivalent to solving the delegated game for x^{c} (Proposition 1). Hence, the two games yield the same equilibrium prices and quantities, i.e. $p_i(\mathbf{x}^{\mathrm{d}}) = p_i(\mathbf{x}^{\mathrm{c}})$ and $q_i(\mathbf{x}^{\mathrm{d}}) = q_i(\mathbf{x}^{\mathrm{c}})$ for $i = 1, 2$. Recalling that w^{d} is continuous and monotone increasing in δ, whereas w^{c} is invariant with δ, gives Lemma 3-(*i*). ■

D.3.2 Equal Delegated and Non-Cooperative Welfare Frontier $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$

$(\beta = \widetilde{\beta})$

We know from Proposition 3 that $w^{\mathrm{d}} = w^{\mathrm{n}}$ for $\delta = 0$.

So as to compare welfare for the other values of β, we need to establish some results along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$.

i) We show that consumer surpluses and gross concentrated profits (i.e., before R&D costs) are the same in the delegated and non-cooperative games along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$. Recall that by definition $x^{\mathrm{d}}=x^{\mathrm{n}}$ along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$. Therefore production costs, together with quantities and thus prices, are identical in the two games, that is $c_i(\mathbf{x}^{\mathrm{d}})=c_i(\mathbf{x}^{\mathrm{n}})$, $p_i(\mathbf{x}^{\mathrm{d}})=p_i(\mathbf{x}^{\mathrm{n}})$, and $q_i(\mathbf{x}^{\mathrm{d}})=q_i(\mathbf{x}^{\mathrm{n}})$, for $i=1,2$. Hence gross profits and consumers' surplus are equal in the delegated and non-cooperative games along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$.

ii) We show that the sign of the difference between total R&D costs in the delegated and non-cooperative games along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ depends on the sign of δ. We know that $-\delta x_1x_2 \overset{\leq}{\underset{>}{=}} 0$ if and only if $\delta \overset{\geq}{\underset{<}{=}} 0$, for all $x_1,x_2>0$. Then recall from Lemma 1-(*ii*) that $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} \overset{\geq}{\underset{<}{=}} 0$ if and only if $\beta \overset{\leq}{\underset{>}{=}} \widetilde{\beta}$. It follows that:

$$-\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}x_1x_2 \overset{<}{\underset{>}{=}} 0 \quad \text{if and only if} \quad \beta \overset{<}{\underset{>}{=}} \widetilde{\beta}, \tag{D.19}$$

for all $x_1,x_2>0$. Hence, (D.19) means that along $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ the laboratory's costs are less than (equal to, greater than) firms' total in-house R&D costs if and only if β is less than (equal to, greater than) $\widetilde{\beta}$.

As welfare is the sum of consumer surplus and firms' net profits, it follows from i) and ii) that:

$$w^{\mathrm{d}}\Big|_{\delta=\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}} \overset{>}{\underset{<}{=}} w^{\mathrm{n}} \quad \text{if and only if} \quad \beta \overset{<}{\underset{>}{=}} \widetilde{\beta}. \tag{D.20}$$

Define:

$$\widehat{\Delta}(\delta)=w^d-w^n,$$

and use (D.20) to establish the existence of $\delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$ for $\beta<\widetilde{\beta}$, and $\beta>\widetilde{\beta}$ respectively.

$(\beta<\widetilde{\beta})$

Claim A: $\widehat{\Delta}(\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}) > 0$.

This claim follows from (D.20). Moreover $\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} < \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$ from Lemma 2-(*ii*), for $\beta < \widetilde{\beta}$.

Claim B: $\widehat{\Delta}(0) > 0$.

Recall that $w^{\mathrm{c}} = w^{\mathrm{d}}$ for $\delta = 0$ by Proposition 1, and $w^{\mathrm{c}} < w^{\mathrm{n}}$ for $\beta < \widetilde{\beta}$ from Hinloopen (2000). Claim B follows by transitivity.

$(\beta > \widetilde{\beta})$

Claim C: $\widehat{\Delta}(\delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}) < 0$.

The proof of claim C follows directly from (D.20).

Claim D: $\widehat{\Delta}(\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}) > 0$.

On the firms' side, by definition $\pi^{\mathrm{d}} = \pi^{\mathrm{n}}$ along $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$. On the consumers' side, because $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$ from Lemma 2-(*ii*), the following holds along $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$: $x^{\mathrm{d}} > x^{\mathrm{n}}$, and consequently $c_i(\mathbf{x}^{\mathrm{d}}) < c_i(\mathbf{x}^{\mathrm{n}})$ which in turn leads to $p_i(\mathbf{x}^{\mathrm{d}}) < p_i(\mathbf{x}^{\mathrm{n}})$, and $q_i(\mathbf{x}^{\mathrm{d}}) > q_i(\mathbf{x}^{\mathrm{n}})$, for $i = 1, 2$. On the laboratory's side, we know that $\delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}} > \delta_{\mathcal{L}=0}$ from Lemma 2-(*ii*) in the case of positive direct externalities, which implies that $\mathcal{L} = 0$ from Proposition 2. As both firms and consumers are better-off in the delegated R&D game than in the non-cooperative one, while the laboratory earns zero benefits in either game, claim D is true.

Claims A to D, Lemma D-3, and that w^{n} does not vary with δ, combine to give Lemma 3-(*ii*). ■

All four figures are drawn for $a = b = 1$, $c = 3/4$, $\gamma = 2$, $\theta = 3/4$. Figures 1, 2, 3 include a reference to the following results by Hinloopen (2000): (*i*) $x^{\mathrm{c}} \gtreqless x^{\mathrm{n}}$ if and only if $\beta \gtreqless \theta/(2-\theta^2)$; (*ii*) $\pi^{\mathrm{c}} = \pi^{\mathrm{n}}$ at $\beta = \theta/(2-\theta^2)$, otherwise $\pi^{\mathrm{c}} > \pi^{\mathrm{n}}$; and (*iii*) $w^{\mathrm{c}} \gtreqless w^{\mathrm{n}}$ if and only if $\beta \gtreqless \theta/(2-\theta^2)$.

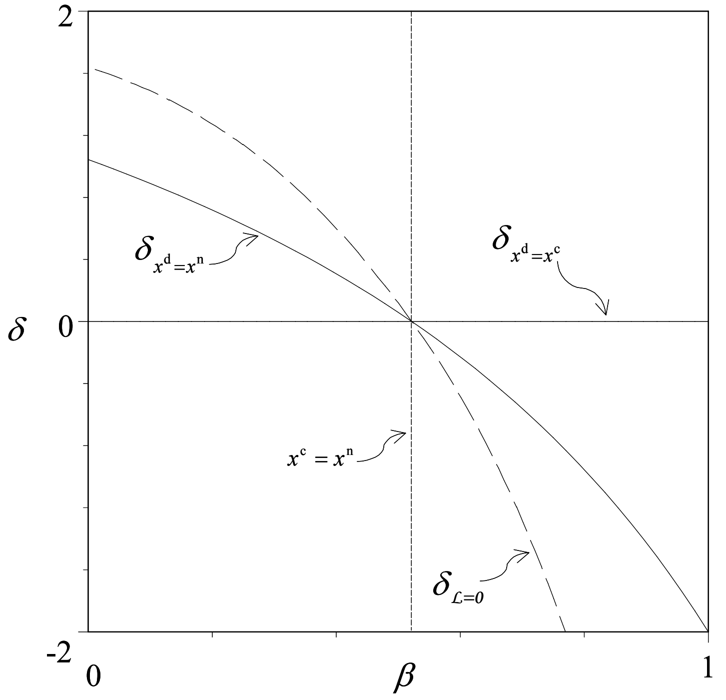

Figure 1: **(R&D outcomes):** $x^{\mathrm{d}} \gtreqless x^{\mathrm{n}}$ if and only if $\delta \gtreqless \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}}$, and $x^{\mathrm{d}} \gtreqless x^{\mathrm{c}}$ if and only if $\delta \gtreqless \delta_{x^{\mathrm{d}}=x^{\mathrm{c}}} = 0$.

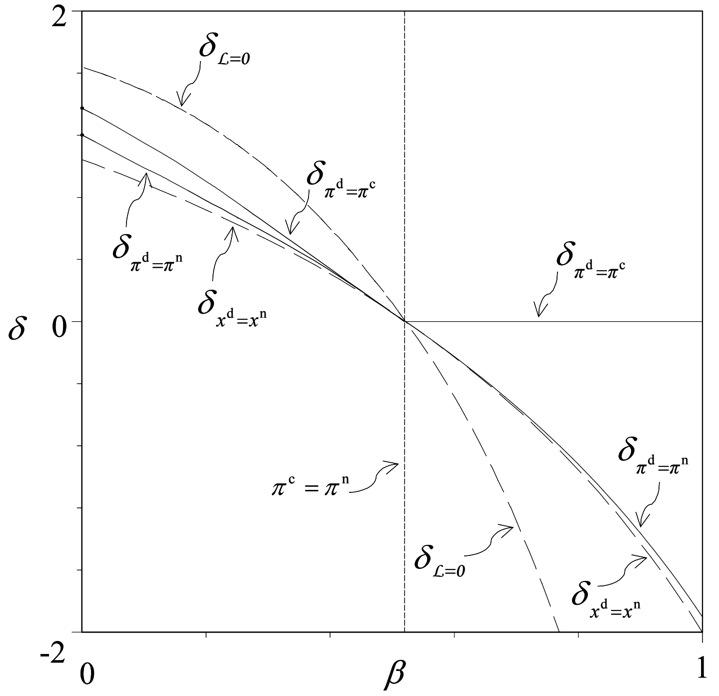

Figure 2: **(Firms' profits):** $\pi^{\mathrm{d}} \gtreqless \pi^{\mathrm{n}}$ if and only if $\delta \gtreqless \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{n}}}$, and $\pi^{\mathrm{d}} \gtreqless \pi^{\mathrm{c}}$ if and only if $\delta \gtreqless \delta_{\pi^{\mathrm{d}}=\pi^{\mathrm{c}}}$.

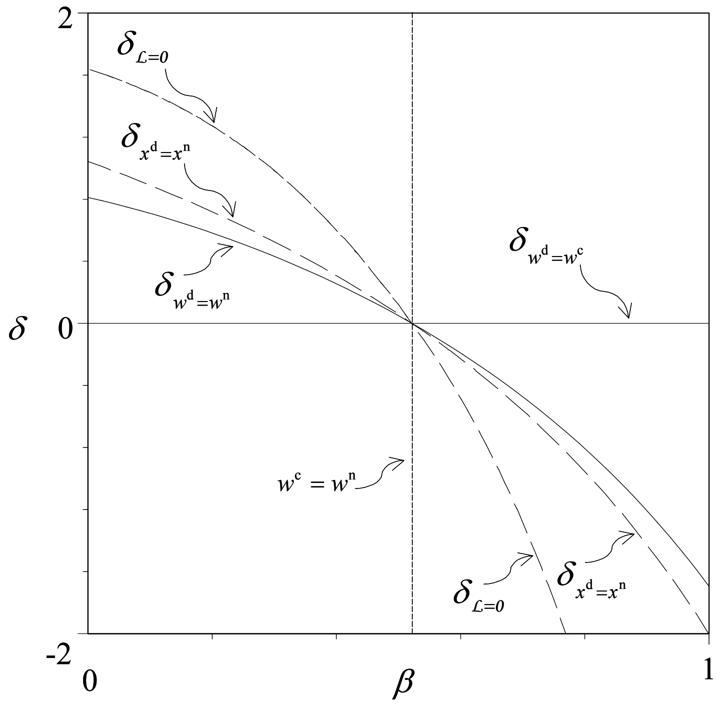

Figure 3: **(Social welfare):** $w^{\mathrm{d}} \substack{>\\=\\<} w^{\mathrm{n}}$ if and only if $\delta \substack{>\\=\\<} \delta_{w^{\mathrm{d}}=w^{\mathrm{n}}}$, and $w^{\mathrm{d}} \substack{>\\=\\<} w^{\mathrm{c}}$ if and only if $\delta \substack{>\\=\\<} \delta_{w^{\mathrm{d}}=w^{\mathrm{c}}}$.

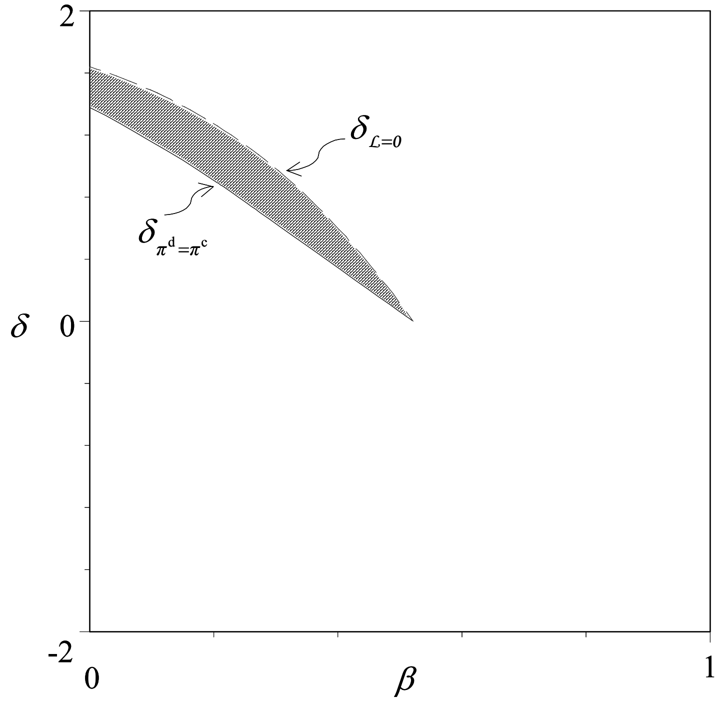

Figure 4: **(Delegation dominance):** the shaded area represents the set of points (β, δ) for which the delegated R&D game is a Pareto optimal organizational form of R&D, and the laboratory earns positive benefits.

References

Aghion, Philippe, and Jean Tirole (1994) 'On the management of innovation.' *Quarterly Journal of Economics* 109(4), 1185–1290

Ambec, Stefan, and Michel Poitevin (2001) 'Organizational design of R&D activities.' Technical Report 38, CIRANO, Montreal, Quebec, Canada

Amir, Rabah, Igor Evstigneev, and John Wooders (2003) 'Noncooperative versus cooperative R&D with endogenous spillover rates.' *Games and Economc Behavior* 42(2), 183–207

Bernheim, B. Douglas, and Michael D. Whinston (1986) 'Menu auctions, resource allocation, and economic influence.' *Quarterly Journal of Economics* 101(1), 1–32

Billete de Villemeur, Etienne, and Bruno Versaevel (2003) 'From private to public common agency.' *Journal of Economic Theory* 111(2), 305–309

Crémer, Jacques, and Michael H. Riordan (1987) 'On governing multilateral transactions with bilateral contracts.' *RAND Journal of Economics* 18(3), 436–451

d'Aspremont, Claude, and Alexis Jacquemin (1988) 'Cooperative and noncooperative R&D in duopoly with spillovers.' *American Economic Review* 78(5), 1133–1137

Hinloopen, Jeroen (2000) 'Strategic R&D cooperatives.' *Research in Economics* 54(2), 153–185

Howells, Jeremy (1999) 'Research and technology outsourcing.' *Technology Analysis and Strategic Management* 11, 17–29

Howells, Jeremy, Andrew James, and Khaleel Malik (2003) 'The sourcing of technological knowledge: distributed innovation processes and dynamic change.' *R&D Management* 33(4), 395–409

Kamien, Morton I., Shumel S. Oren, and Yair Tauman (1992) 'Optimal licensing of cost-reducing innovation.' *Journal of Mathematical Economics* 21(5), 483–508

Katz, Michael L., and Carl Shapiro (1986) 'How to license intangible property.' *Quarterly Journal of Economics* 101(3), 520–567

Laffont, Jean-Jacques, and David Martimort (1997) ‘The firm as a multicontract organization.’ *Journal of Economics and Management Strategy* 6(2), 201–234.

Laussel, Didier, and Michel Le Breton (2001) ‘Conflict and cooperation: The structure of equilibrium payoffs in common agency.’ *Journal of Economic Theory* 100(1), 93–128

Martimort, David, and Lars Stole (2001) ‘Contractual externalities and common agency equilibria.’ Working paper 581, CESifo

— (2003) ‘Market participation under delegated and intrinsic common agency games.’ Université de Toulouse and University of Chicago, Mimeo

Martin, Stephen (2001) ‘Strategic research partnerships: Evidence and analysis.’ In *Strategic Research Partnerships: Proceedings from an NSF Workshop,* ed. Nicholas S. Vonortas John E. Jankowski, Albert N. Link National Science Foundation, Division of Science Resources Studies National Science Foundation 4201 Wilson Blvd., Suite 965, Arlington, VA 22230. http://www.nsf.gov/sbe/srs/nsf01336/p1s1.htm

Milgrom, Paul, and John Roberts (1990) ‘The economics of modern manufacturing: technology, strategy, and organization.’ *American Economic Review* 80(3), 511–528

Singh, Nirvikar, and Xavier Vives (1984) ‘Price and quantity competition in a differentiated duopoly.’ *RAND Journal of Economics* 15(4), 546–554

Vencatachellum, Désiré, and Bruno Versaevel (2004) ‘R&D delegation in a duopoly with spillovers.’ HEC Montreal, Université de Montreal

Notes

[1] In this paper, outsourcing refers to firms delegating R&D services through a formal contract which specifies the required services and the associated payments. These R&D services can alternatively be done in-house by the firms. Detailed descriptions and examples appear in Howells (1999).

[2] The business literature (Howells, James and Malik 2003, for example) opposes the outsourcing of new technological knowledge, under conditions stipulated in a contract agreed beforehand, to firms licensing existing technological knowledge.

[3] These numbers refer to industrial R&D performed outside a company's facilities and funded from all sources except the Federal Government. For details and distributions by industry and by size of company, see NSF (2003, Table A-10, pp. 46-47).

[4] Dobler and Burt (1996) observe that "R&D services normally are purchased through one of the two methods of compensation: a fixed price for a level of effort (e.g., fifty days) or a cost plus fixed or award fee" (p. 416).

[5] For details on these contracts, and other examples, see: www.recap.com/bday.nsf.

[6] Other theoretical studies also consider an intermediate R&D market, but with a different focus. For example, Aghion and Tirole (1994) examine the allocation of property rights as a solution to contract incompleteness between the R&D and user stages, while Ambec and Poitevin (2001) assume complete contracts and introduce some informational asymmetry. Both of these papers assume one laboratory and one user. We differ in keeping an oligopolistic final market structure, as in the two streams of literature reviewed here, and assume the laboratory can simultaneously contract with more than one firm, as observed in many industries.

[7] Laffont and Martimort (1997) call indirect and direct externalities "type 1" and "type 2" externalities repectively.

[8] We capitalize on recent results on the structure of equilibrium payoffs in common agency by Laussel and Le Breton (2001) and their extension to models with externalities among principals by Billete de Villemeur and Versaevel (2003).

[9] The laboratory bears all R&D costs, while the functional form of firms' net profits in the delegated R&D game is similar to Crémer and Riordan (1987) who model multilateral transactions with bilateral contracts, but with transfer payments that are here contingent on the laboratory's choice of R&D outputs.

[10] The proofs of the existence and stability properties, that characterize truthful Nash equilibria for a class of common agency games which include the present specification, are in Bernheim and Whinston (1986).

[11] Lemma 1-(*ii*) implies that $\delta_{\check{x}_j=\hat{x}_j=0} = \delta_{x^{\mathrm{d}}=x^{\mathrm{n}}} < \delta_{\mathcal{L}=0}$.