The Hierarchical Firm Resolves Empty Cores: A Coasean Model and Two Examples

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Abstract

This note proposes a clarification of the definition of the Coasean hierarchical firm. A simple model in the theory of the core is constructed that formalizes the optimization problem underlying the industrial organization *à la* Coase. Two simple specific examples illustrate the proposition that empty core problems are more fundamental determinants of the real-world organization of production than the various phenomena that can lead to them, including "transaction costs" and "social costs". In the two cases, the Coasean firm is rationalized as an institutional remedy that resolves an empty core problem by eliminating a selection of potential market transactions. It does not form if the core in non empty, if an empty core is not resolved, or if it is resolved by some alternative device that does not resort to authority of command.

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INTRODUCTION

Coase (1937)'s evergreen insight is that the observed industrial organization of production is the result of an optimization problem, that operates a selection of competing devices for the co-ordination of transactions. On the market, transactions get allocated by the price mechanism. In firms, resources are directed by hierarchical authority. In Cheung (1983)'s words:

> Coase's central thesis is that differences in the costs of operating institutions (transaction costs) lead to the emergence of a firm to supersede a market. On the one hand, market transactions involve products or commodities; on the other, "firm transactions" involve factors of production. The growth of a firm may then be viewed as the replacement of a product market by a factor market, resulting in a saving in transaction costs. This thesis is not easy to understand because Coase does not define "the firm"; nor (...) is there a clear distinction between a product market and a factor market. (p. 1)

Surprisingly, many years later, the definition of the Coasean firm, of what it exactly is, and what it is not, remains unclear. In a recent introductory paper of a special issue on theories of the firm, Garrouste and Saussier (2005) emphasize that "[*t*]*he definition of the firm*, viewed as the place where the coordination through prices is replaced by the coordination through authority, is vague" (p. 180). They then call for more research efforts on that issue.

The objective of this note is to contribute to the clarification of Coase's central thesis in the simplest possible analytical treatment of a firm formation in a market economy. Toward this aim, a basic cooperative model is described in section 1, in which potential coalitions compete for the use of inputs owned by individual agents in order to transform them into a single output. In the words of Cheung, these inputs can be considered either as "products" (when exchanged on the market), or as "factors" (when transformed by coalitions). Two specifications, or "examples", are derived in turn from this formal setting. In section 2, the first example focuses on transaction costs, namely bargaining problems and managerial mistakes. In section 3, another example concentrates on social costs, that is negative externalities in production. In the two cases, the Coasean hierarchical firm is introduced as a means to resolve an empty core, by restricting competition for inputs among potential coalitions, and thereby to make group rationality prevail over individual rationality. Section 4 concludes the note.

1. THE BASIC SET-UP

Consider an economy with m inputs in $\mathbf{x} = (x_1, \ldots, x_i, \ldots, x_m)$, indexed by i in $M = \{1, \ldots, m\}$, that can be transformed in quantities y of a single output defined as a *numéraire*. This means that all input prices represented in $\mathbf{w} =$

$(w_1, \ldots, w_i, \ldots, w_m)$ are normalized to be expressed relative to the price of the output, which is 1. Individual agents, indexed by j in $N = \{1, \ldots, n\}$, are endowed with the inputs. Each agent j owns $\mathbf{x}^{\{j\}} = (x_1^{\{j\}}, \ldots, x_i^{\{j\}}, \ldots, x_m^{\{j\}})$, such that $\sum_{j \in N} x_i^{\{j\}} = x_i$, all i. Inputs can either be sold on the market in exchange of the output, or transformed in order to produce the final output. This can be done by forming coalitions $S \in 2^N \backslash \{\emptyset\}$, with $\#S = s \leq n$. The coalitions compete for the inputs owned by the agents. This competition determines input prices. With each coalition S is associated a non-empty production set $Y^S \subset R_+^m \times R_+^1$. The same technology is available to all coalitions. It is described by the production function $f : R_+^m \to R_+^1$ such that $y = f(\mathbf{x}^S)$, defined as the maximum level of output that can be obtained from the transformation of $\mathbf{x}^S = (x_1^S, \ldots, x_m^S)$, where $x_i^S = \sum_{j \in S} x_i^{\{j\}}$, all i (note that $x_i^N = x_i$). For each coalition, the production possibility set is $Y^S = \{(\mathbf{x}^S, y) : y \leq f(\mathbf{x}^S)\}$. This economy is represented hereafter by the list of data $E = (m, n, \mathbf{x}, \{\mathbf{x}^{\{j\}}\}_{j \in N}, \{Y^S\}_{S \subseteq N})$. In the following sections, distinct specifications are introduced in the "transaction costs" and "social costs" cases, respectively.[1]

2. THE CASE OF TRANSACTION COSTS

The first contribution by Coase (1937) to the theory of the firm starts by referring to "the economic system as it is normally treated by the economist" of that time [Coase (1937, p. 387)]. This benchmark situation can be defined precisely as a static efficient competitive equilibrium, that is an allocation that belongs to the *core* of the economy. In the present set-up, applying the theory of the core consists in a systematic examination of all $2^n - 1$ possible patterns of trade among (coalitions of) individuals. Competition is understood as a (re)contracting process toward the selection of a set of allocations that cannot be improved upon at the advantage of some (coalition of) individual(s) without making at least one individual worse off. In $E = (m, n, \mathbf{x}, \{\mathbf{x}^{\{j\}}\}_{j \in N}, \{Y^S\}_{S \subseteq N})$, each coalition S (including a singleton $\{j\}$ and N itself) of individual owners of $\mathbf{x}^S$ can either sell inputs (i.e., "products" here) and obtain $\mathbf{w}\mathbf{x}^S$, or transform inputs (i.e., "factors") and obtain $f(\mathbf{x}^S)$. The most profitable alternative that is chosen is then $\sup\{\mathbf{w}\mathbf{x}^S, f(\mathbf{x}^S)\}$. The conditions for an allocation to be in the core are: (*i*) there exists a solution in prices $\mathbf{w}^*$ (where the superscript denotes an equilibrium solution) such that no coalition can obtain a strictly positive net profit by transforming inputs, and (*ii*) this solution $\mathbf{w}^*$ is feasible in the sense that the total gains to all agents are equal to the maximum output the largest coalition could produce. This translates into the conditions

$$\mathbf{w}\mathbf{x}^S \geq f(\mathbf{x}^S) \text{ and } \mathbf{w}\mathbf{x}^N = f(\mathbf{x}^N), \tag{1}$$

for all $S \subseteq N$. The series of inequalities says that no coalition S (including singletons $\{j\}$ and N) producing $f(\mathbf{x}^S)$ can obtain a higher revenue than the one obtained at equilibrium in the core. The right-hand side equality says that, in the core, the maximum total revenue $f(\mathbf{x}^N)$ to be feasible by coalition N is entirely distributed to input sellers.

This benchmark model can now be completed by the introduction of transaction costs, on the Coasean grounds that it “gives a very incomplete picture of our economic system” [Coase (1937, p. 387)]. This can be done by adding the following conditions: each coalition must pay (*i*) a nonnegative amount g should it decide to transform inputs instead of selling them, and (*ii*) a nonnegative amount h should it decide to sell. The objective is to rationalize the observed division of resource transfers among two “alternative methods of co-ordinating production” [Coase (1937, p. 388)]: markets, on which transactions are organized by the price mechanism; and coalitions, within which transactions are organized by some managerial device. Then the costs of organizing transactions, either by the price-mechanism or by managerial means, are compared. The terms g and h represent transaction costs within the coalitions (say, managerial mistakes), and on the market (say, bargaining problems), respectively. Assuming that “as each producer expands he becomes less efficient” and “the additional costs of organizing extra transactions increase” [Coase (1937, p. 396)], then transaction costs of the managerial kind should be modelled as an increasing function of some measure of the size of productive operations. In like manner, because “the costs of negotiating and concluding a separate contract for each exchange transaction which takes place on a market must (...) be taken into account” [Coase (1937, pp. 390-1)], the costs incurred by bargaining problems can be formalized as an increasing function of the number of inputs supplied to all possible coalitions by individual owners. Accordingly, let g and h be increasing functions of s (a coalition size) and m (number of traded inputs), respectively. The simplest possible forms are $g(s) \equiv \alpha s$ and $h(m) \equiv \beta m$, where α, β are positive parameters. Then the economy is said to have an ϵ-core when there exists a set of input prices $\{w_i\}_{i=1}^m$ which satisfy

$$\mathbf{w}\mathbf{x}^S \geq f(\mathbf{x}^S) - \epsilon(m,s) \ \text{ and } \ \mathbf{w}\mathbf{x}^N = f(\mathbf{x}^N) - \epsilon(m,n), \tag{2}$$

for all $S \subseteq N$, where $\epsilon(m,s) \equiv g(s) - h(m)$ can be negative. Otherwise, the ϵ-core is empty.[2] In that case, let us introduce the specific nature of the Coasean firm as a restriction on the choice of contracts by self-interested agents. In formal terms, this is obtained by the elimination of some of the $2^n - 1$ constraints for a solution to exist. This is tantamount to ruling out the formation of particular coalitions S, for the division of a productive outcome $f(\mathbf{x}^S)$, that individual input owners could obtain by contracting among themselves, exclusively. To see that, consider the following example.

Example 1: In an economy $E = (1, 3, 3, \{x^{\{j\}}\}_{j\in\{1,2,3\}}, \{Y^S\}_{S\subseteq\{1,2,3\}})$, the distribution of inputs is $\{x^{\{j\}}\} = \{1, 1, 1\}$, and the production function is $f(x^S) = x^S - 1$. First, assume that there are no transaction costs. A non-empty ϵ-core (with $\epsilon(m,s) = 0$) is obtained if and only if one can find a price w^* such that $w^* \geq 0$, and $2w^* \geq 1$, and $3w^* = 2$. In this case, the ϵ-core is non-empty, since $w^* = \frac{2}{3}$ satisfies all constraints. Second, assume that there are transaction costs. Recall that $g(s) = \alpha s$ and $h(m) = \beta m$, $\alpha, \beta \geq 0$. With $m = 1$ and $n = 3$, and because of the uniform distribution of input endowments, the system of conditions for the ϵ-core to be non-empty is

$$\begin{cases} w - \beta \geq -\alpha, \\ 2w - \beta \geq 1 - 2\alpha, \\ 3w - \beta = 2 - 3\alpha, \end{cases} \tag{3}$$

where the first two lines correspond to 1-agent and 2-agent coalitions, respectively. The third line results from the condition that, in the ϵ-core, payments $3w$ ($= \sum_{j=1}^{3} wx^{\{j\}}$) *minus* "marketing" costs β cannot be less than the output obtained by the 3-person coalition, together with the condition that, in the ϵ-core, input sales cannot exceed the maximum total output 2 ($= f(x^N)$) *minus* "managerial" costs 3α. Accordingly, a solution in w must satisfy $w = \frac{1}{3}(2+\beta) - \alpha$. Then one finds that the ϵ-core is non-empty for $\beta \leq 1$ (i.e., when transaction costs on the market are low), and empty otherwise (i.e., when transaction costs on the market are relatively high). In the latter case, two possibilities arise:

- the ϵ-core is not resolved: this is obtained if no restriction is imposed on the contracting process. In that case, the largest coalition is not stable. More precisely, 1-person and 2-person coalitions can do better for their members than the 3-person coalition, that is $w^* - \beta < -\alpha$ and $2w^* - \beta < 1 - 2\alpha$, where $w^* = \frac{1}{3}(2 + \beta) - \alpha$, for all $\alpha \geq 0$ and $\beta > 1$. Then the agents obtain $1 - 3\alpha$ as a total gain, which is the sum of a 1-person coalition's revenue $f(x^{\{j\}}) - \alpha$ and of a 2-person coalition's revenue $f(x^{\{k,l\}}) - 2\alpha$, for all distinct j, k, l in N.

- the ϵ-core is resolved: this can be obtained by eliminating potential contracts that involve up to two agents.[3] In that case, there always exists one solution $w^* = \frac{1}{3}(2 + \beta) - \alpha$, for all α and $\beta > 1$. Then the three agents divide total gains equal to $2 - 3\alpha$. This is more than the total revenues obtained when the ϵ-core is not resolved (i.e., $1 - 3\alpha$), and less than the efficient outcome obtained in the same economy without transaction costs (i.e., $f(x^N) = 2$).

As these two alternatives stem from formal conditions that involve a high level of abstraction, we do not want to overemphasize the message they offer to the economic historian of the institutions of production. Observe simply that, when the ϵ-core is empty, the sub-optimal joint revenues result from productive activities that are deliberately conducted in two small coalitions (in the sense that $s < n$). By contrast, when the ϵ-core is resolved, larger revenues are obtained by arbitrarily eliminating the potential contracts that prevent production to occur in the grand coalition ($s = n$). The comparison of the two situations is of interest. In the former case, a coalition represents some non hierarchical form of production, say a producer co-operative, in which no centralized authority of command appears. This is not a

Coasean firm. To fix ideas, it can be referred to as a "family" in opposition to the grand coalition, obtained by the restriction of competition among potential coalitions for the inputs owned by individual agents, which is a "hierarchy".[4] Indeed, in the latter case, the elimination of potential contracts is a formal way of introducing the authority of command embodied in the entrepreneur, defined as "the person or persons who, in a competitive system, take the place of the price mechanism in the direction of resources" [Coase (1937, p. 388n)]. This yields the Coasean firm.[5] □

This example sharpens the Coasean lesson. To put it in stark terms, the elimination of possible transactions is the hallmark of the hierarchical firm, a co-ordination method that supersedes the market. In the presence of transaction costs, the introduction of authority enhances the level of total gains. These gains fall short of what would be obtained in a world of zero transaction costs.

3. THE CASE OF SOCIAL COSTS

Another well-known contribution by Coase (1960) focuses on situations characterized by negative externalities in production. The red thread of the analysis describes two agents who impose harmful effects on others. The problem under scrutiny is sketched out in the form of a bucolic example, involving a farmer and a cattle-raiser operating on neighboring properties. Straying cattle destroy crops and, without any fencing between the properties, an increase in the size of the cattle-raiser's herd happens to increase the total damage to the farmer's crop. In substance, the so-called Coase theorem asserts that, in the absence of transaction costs, a negotiation can lead to a mutually satisfactory agreement which maximizes wealth. More precisely, for any possible initial distribution of property rights over productive inputs (i.e., endowments in steers and acres), side payments can bring about the same Pareto optimal value of total production (i.e., obtained from the herd and from the field in this case) as would have been obtained by a single owner of all inputs engaged in the two activities. However, with more than two agents, Aivazian and Callen (1981) show that the Coase theorem does not always apply. They offer an example with three agents (two factories that pollute a laundry) in which no stable Pareto-optimal agreement can be reached by the negotiating parties. This implies that the core is empty. The authors conclude that

> [e]mpirically, however, it is difficult to differentiate between contractual arrangements that arise because of the nonexistence of the core from those that arise from transaction costs when there is a core. (...) No one knows what actually happens when the core is empty. Do the participants agree to a Pareto-optimal solution or will they merely stop negotiating? [Aivazian and Callen (1981, p. 181)]

In a comment, Coase (1981) does not address the question. He rather explains why the particular example by Aivazian and Callen has not led him to modify his views. To do so, he concentrates on an alternative interpretation of what would happen in the postulated conditions of the given example, with no attempt to generalize the argument to other possible examples of negotiations for which the core is empty. Moreover, as noted by Telser (1997, p. 11n), Coase (1988)'s most elaborate exposition of his theorem does not refer to the problem posed by an empty core. Arguably, the question can be examined in the terms proposed in the previous section: if agents stop negotiating, production by small coalitions leads to suboptimal total gains; if the core is resolved by eliminating some possible contracts, a hierarchical firm is formed that leads to a Pareto-optimal solution. The latter firm can thus be seen as a specification of what Aivazian and Callen (2003) refer to, in very general terms, as "contractual arrangements" in which parties may participate to "*optimally force* a solution on the bargaining process" (pp. 292-3, added emphasis) when the Coase theorem breaks down. Here again, a specific example helps clarifying the argument.

Example 2: An economy $E = (1, n, x, \{x^{\{j\}}\}_{j \in N}, \{Y^S\}_{S \subseteq N})$ is such that

$$f(x^S) = \begin{cases} x^S - x^N & \text{if} \quad s < n, \\ -x^N & \text{if} \quad s = n, \end{cases} \tag{4}$$

where s is the size of a coalition S. This is a pure case of negative externalities, since only "bads" are produced.[6] Each coalition S corresponds to a quantity of inputs $\mathbf{x}^S$ under common ownership. A small coalition (i.e., $s < n$) produces the means of limiting the level of externalities (say, smoke) it receives from others. This does not hold for the grand coalition (i.e., $s = n$). Let $i, j \in N$ with $i \neq j$ throughout. For the sake of simplicity, assume that the initial allocation is uniform, with $x^{\{j\}} = 1$, all j. Consider the cases $n = 2$ and $n > 2$ in turn. If $n = 2$, there is a non-empty core if a price w^* can be found such that $w^* \geq -1$, and $2w^* = -2$, which holds if and only if $w^* = -1$ (this is a negative number, because what is produced is a nuisance in this example). A non-empty core describes mutually satisfactory side payments that distribute Pareto-optimal total gains among agents included in the largest possible coalition. This is the outcome predicted by the Coase theorem, and no (Coasean) firm was required to obtain it. If $n > 2$, for S of size $s = n - 1$, the condition for individual rationality imposes $(n-1)w \geq -1$. Since there are n such inequalities, adding them yields $n(n-1)w \geq -n$. Moreover, the feasibility condition requires that $nw = -n$. One can eliminate w by plugging the latter expression into the former one. This gives the necessary condition $-n(n-1) \geq -n$, i.e., $n \leq 2$. In words, if there are strictly more than two agents in the economy, the core is empty. This result leads us to establish a link with the previous example. Indeed, here again, one can resolve the empty core by eliminating potential contracts. This is exactly what a Coasean firm does. To see this, consider the case $n = 3$. The core is non-empty if and only if there is a price w such that

$$\begin{cases} w \geq -2, \\ 2w \geq -1, \\ 3w = -3, \end{cases} \tag{5}$$

where the first two lines correspond to 1-agent and 2-agent coalitions, respectively. The third line expresses the condition that, in the core, an imputation $3w$ ($= \sum_{j=1}^{3} wx^{\{j\}}$) cannot be improved by the 3-person coalition, together with the condition that the total payment to the inputs cannot exceed the maximum total output to be feasible -3 ($= f(x^N)$). In this case, the core is empty, since one cannot find a price so that every pair (i, j) obtains at least -1 as an outcome. However, by eliminating the potential contracts that involve exactly two agents, one resolves the empty core. Note that, in contrast to what would have been obtained in the presence of transaction costs (see the previous example), this leads to an allocation that maximizes the value of total production, that is $f(x^N) = -3$. Individual gains are -1 for each agent.[7] □

In this example, up to $n = 2$, the absence of transaction costs, when the core is non-empty, implies that changes in the distribution of property rights affect individual gains only. The proposition can then be extended to the case $n \geq 3$, when the core is empty, by resorting to authority (i.e., the hierarchical firm) as introduced formally in the previous section. Rather than limiting the scope of the Coase theorem, this interpretation reinforces the relevance of the claim that the firm, in practice, can be understood as a hierarchical device that supersedes the market for efficiency reasons.

4. CONCLUSION

Coase neglected the examination of contracting situations characterized by an empty core. By focusing on the latter cases, the simple formalization proposed in this note has sharpened the Coasean message by emphasizing the link between the papers published in 1937 and in 1960, and by pointing to the consistency of their respective contributions to the theory of the firm. To summarize, Coase's hierarchical firm appears as what eliminates potential market transactions by pooling a set of productive resources under common ownership to resolve an empty (ϵ-)core. This may hold in the presence of transaction costs (example 1) or (exclusively so) of social costs (example 2). In the latter case only, Pareto efficiency is obtained. The implication is that empty core problems may be more fundamental determinants of the real-world industrial organization of production than the manifold phenomena that can lead to them, of which transaction costs and social costs are examples, among others.[8]

There is at least one major criticism that may be levelled against the present paper: the elimination of possible coalitions in the examples is too drastic. Indeed, contractual arrangements can be imagined that would penalize sufficiently the relevant coalitions toward a resolved core. However, the chosen research strategy is sophisticated enough to put forth two interesting propositions in a self-contained representation of a competitive economy. Firstly, the exercise of authority (i.e., the restriction on the choice of contracts) is not assumed to be the privilege of a representative entrepreneur (the boss) over others (subordinates) in a *given* firm. Authority is introduced as what *establishes* the firm in order to make group rationality prevail over individual rationality. This says what the Coasean firm is. Secondly, the hierarchical firm comes up with clear boundaries (i.e., the set of resources in the coalition that forms as a result of the exercise of authority) that cannot be obtained in all versions of the model (or "examples"). It does not form if the core in non empty, if an empty core is not resolved, and if an empty core is resolved by some alternative device that does not resort to authority of command. This says what the Coasean firm is not.

NOTES

[1] As far as technical specifications are concerned, this model is adapted from a much more general study of competition with the help of core theory by Telser (1996), who offers necessary conditions on the production function f for a non-empty core.

[2] On the ϵ-core solution concept (or quasicore, as it is sometimes called), together with related interpretations, see Shubik (1984). For a survey of applications of the ϵ-core solution concept to topics in industrial organization, see Reid (1987). The ϵ-core is the set of efficient payoff vectors that cannot be improved upon by any subcoalition, when coalition formation entails a "cost" (or a "bonus" when ϵ is negative), which may depend on the number of individuals in a coalition, or take the form of a fixed charge. This offers a very natural way of introducing frictions (or oil) in the coalition formation mechanism.

[3] The elimination of six possibilities out of seven ($= 2^3 - 1$) is a very drastic measure. This particular case is rooted in the uniform distribution of endowments (chosen for the sake of clarity). A more inegalitarian distribution would not necessarily result in the same treatment for all singletons and two-party coalitions. Consider an alternative distribution, say $x^{\{1\}} = 1$, $x^{\{2\}} = 1$ and $x^{\{3\}} = 2$, with $\alpha = 3$ and $\beta = 2$. Then one resolves the empty core by eliminating only three possible coalitions (i.e., $S = \{3\}$, $S = \{1, 3\}$, and $S = \{2, 3\}$).

[4] The term "hierarchy" is used in the well-known sense of the early contributions by Williamson (1967, 1975).

[5] The two cases (empty core *vs.* resolved core) also connect to Leijonhufvud (1986)'s distinction between "crafts production" and the "factory system". The former institutional form of production is best illustrated by the following example that relates to our empty core case: "(...) the organization in [the fourteenth-century arsenal of Venice] was not that of a single firm; instead, numerous craftsmen, owning their own tools, each with a few journeymen and apprentices, operated within the arsenal and cooperated via exchange transactions in the building and outfitting of ships. In short, the famous arsenal was not a factory and not a firm." [Leijonhufvud (1986, p. 204)]. When the empty core is resolved, the proposed characterization of the Coasean firm evokes the factory system, in which "a firm is formed and any capitalist who joins has to give up ownership of his machines and accept shares in the firm" [Leijonhufvud (1986, p. 212)].

[6] This example is adapted from the "garbage game" described by Shapley and Shubik (1969, pp. 681-2). In the latter contribution, each agent has exactly 1 bag of garbage which he must dump in someone's yard. The utility of having b bags is $-b$. Agents are in $N = \{1, 2, \dots, n\}$, and the characteristic function of the game is $v : 2^N \backslash \{\emptyset\} \rightarrow R^1$. Each coalition $S \subset N$, $S \neq N$, offers the means to leave the bags of garbage "outside the doors of the coalition" (i.e., $v(S) = s - n$ if $s < n$), and the

all-player set cannot avoid “fouling its own nest”, in the words of the authors (i.e., $v(N) = -n$).

[7] In this set-up, the same Pareto-optimal value of total gains is obtained for all possible allocations of endowments. To see that, consider an alternative initial allocation, say $x^{\{1\}} = \frac{1}{4}$, $x^{\{2\}} = \frac{1}{4}$ and $x^{\{3\}} = \frac{5}{2}$. In this case, one resolves the empty core by eliminating one (out of three) 1-agent coalition (i.e., $S = \{3\}$), and two (out of three) 2-agent coalitions (i.e., $S = \{1, 3\}$ and $S = \{2, 3\}$). The value of total production is still -3, and individual gains are $-\frac{1}{4}$, $-\frac{1}{4}$, and $-\frac{5}{2}$, for agents 1, 2, and 3, respectively.

[8] See Aivazian and Callen (2003) for a detailed presentation and discussion of the many possible reasons for which an empty core problem may arise, implying that the Coase theorem breaks down.

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