

Patent Pools and the Dynamic Incentives to R&D¹

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Abstract

Patent pools are cooperative agreements between several patent owners to bundle the sale of their respective licenses. In this paper we analyze their consequences on the speed of the innovation process. We adopt an *ex ante* perspective and study the impact of possible pool formation on the incentives to innovate. Because participation in the creation of a pool acts as a bonus reward on R&D activity, we show that a firm's investment pattern is upward sloping over time before pool formation. The smaller the set of initial contributors, the higher this effect. The *ex ante* perspective leads to policy implications. It reveals that patent pools encourage high R&D efforts, in comparison to a joint profit-maximizing benchmark, by early innovators. Patent pools can thus be used as a stimulating instrument for the generation of the first patents when innovation is sequential in nature. We also find that, when solicited by firms in advance of the pool foundation, a forward-looking regulator has an incentive to delay the clearance date. This can result in a suboptimal pool size, as compared with the social welfare optimum. This questions the relevance of a review process by competition authorities before a pool forms.

Keywords: Licensing, R&D races, Innovation, Competition policy.

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1 Introduction

Patent pools are cooperative agreements among several patent owners to license in a package a group of their respective patents to third parties. Although patent pools have long been suspected of facilitating the implementation of anti-competitive behavior, recent communications by competition authorities recognize the potential virtues of patent pools as a means of improving social welfare. In the *Antitrust Guidelines for the Licensing of Intellectual Property* (1995), patent pools are declared to “provide procompetitive benefits by integrating complementary technologies, reducing transaction costs, clearing blocking positions, and avoiding costly infringement litigation” (p. 28). In substance, by allowing one-stop shopping, the pool gives access to more efficient licensing. It can thus increase the private value of the constitutive patents, and also increase social welfare by facilitating the diffusion of innovations. As a consequence of this more favorable position, patent pools re-emerged in the recent years, mainly in high-technology sectors. Examples include MPEG-2 (1997), MPEG-4 (1998), Bluetooth (1998), DVD-ROM (1998), DVD-Video (1999), and 3G-Mobile Communications (2001). They participate in the diffusion of products and services which are a growing part of everyday life.

There are two possible research viewpoints on questions concerning patent pooling, namely the *ex ante* perspective and the *ex post* perspective.

We adopt the *ex ante* perspective to identify the basic trade-offs that determine the dynamic incentives to perform R&D. We choose to investigate the case in which firms do anticipate the possibility of participating in a patent pool when they contemplate investing in a R&D program, and therefore attempt to patent in clearly identified technological domains toward this aim. As the pooling of patents allows firms to coordinate their licensing behavior, it increases their return on investment. This must have an impact on their investment decisions.

Although legal scholars and business practitioners have documented particularities of all kinds in clinical analyzes of patent pools, a few stylized features characterize recent arrangements:¹ (i) they form at the issue of a voluntary process in which patentees may request a clearance statement from the regulator by submitting a pool proposal;² (ii) they establish a mechanism for dividing among patentees the royalty stream;³ (iii) they evolve over time to incorporate the innovations that are patented after pool creation.⁴ We develop a tractable dynamic model of pool formation that incorporates these features and study some of the consequences of the pool formation mechanism on the speed of innovation.

We find that, compared with the situation where there is no possible pooling, the perspective of participating in the formation of a pool enhances the speed of R&D. As firms value more being among the initial contributors to the arrangement – as opposed to innovating after the formation of the pool and then negotiating entry – we show that the equilibrium pattern of innovative efforts is upward sloping before the formation of the pool. We present a complete comparison of equilibrium R&D effort patterns for two different pool sizes. We show that an increase in the minimal number of patents, as required to form a pool, dilutes firms' incentives

¹See Merges (1999) for a detailed description of the many organizational forms and contractual provisions of past and current pooling arrangements.

²On this see the four U.S. Department of Justice Business Review Letters (1997, 1998, 1999 and 2002, available at: www.usdoj.gov/atr/public/busreview/1170.htm) regarding MPEG-2, the two DVD pools, and the 3G platform, respectively. In the European Union, the regulatory treatment of patent pools is presented in the European Commission's Guidelines on the application of Article 81 of the EC Treaty to technology transfer agreements (2004/C 101/02, available at: <http://eur-lex.europa.eu/en/index.htm>). The guidelines do not require firms to solicit a review process.

³For example, in the MPEG-2 case, the pool initiators licensed their patents to MPEG LA, a licensing agent. This separate entity offers the portfolio of patents as a package to third parties (although any particular patent may be licensed from a pool member individually). Royalties are then distributed according to a formula which reflects the respective weight of patent contributors to the pool. On this see Merges (1999), and Layne-Farrar and Lerner (2006).

⁴The United States Patent and Trademark Office (2000) documents the fact that, in recent cases, the Department of Justice has not only focused on the complementary nature of existing patents to be licensed at the date of pool formation, but also required that participants license to each other complementary patents “they obtain *in the future*” (p. 7, original emphasis). Moreover, new patents in the pool do not necessarily originate from the pool initiators. For example, after 1997, the MPEG-2 pool grew by including additional patents from a set of firms which did not participate in the foundation of the pool, including France Telecom, Hitachi, and JVC.

to obtain the first patents, and thus decreases the equilibrium R&D investments. By using the joint profit-maximizing R&D level as a benchmark, we then obtain that the possibility given to firms to create a pool encourages high investment levels by early inventors. When innovation is cumulative in nature, patent pool arrangements can thus serve as a policy instrument for the generation of the first patents. Eventually, we find that the regulator's review process of pool proposals may induce inefficiencies according to a social welfare criterion. This is because a forward-looking regulator is shown to have an incentive to postpone a previously announced date of pool formation, a time inconsistency result. This leads us to question the relevance of soliciting a review process by competition authorities *before* a pool forms.

By contrast, the *ex post* perspective follows antitrust practices for reviewing the impact of a pool on welfare after the formation of that pool. The objective is then to identify what kind of pools should be authorized by the regulator, a question that is examined by Shapiro (2001). In this pioneering contribution, a very simple model lends theoretical support to the prevalent view that welfare is harmed when patents are perfect substitutes, and raised when patents are perfect complements. However, it is not always obvious that given patents are substitutes or complements. In that case, a relevant objective is to provide the regulator with some means to discriminate among pool candidates. Lerner and Tirole (2004) address this problem in a model that describes the full range between the extreme cases of perfectly substitutable and perfectly complementary patents. In this more general context, they notably show that the requirement that independent licenses be offered by pool members to third parties can be used as a screening device. The reason is roughly that independent licensing is innocuous when patents are complements, but reduces the pool's profits when patents are substitutes. Compulsory independent licensing thus lowers the incentive to pooling substitutable patents.⁵ Lerner, Strojwas, and

⁵Brenner (2004) extends the Lerner-Tirole setup to include a static (i.e., post innovation) modelling of the

Tirole (2007) analyze 63 patent pool agreements. They find empirical support to the conjecture that pools of complementary patents are more likely to authorize independent licensing by member firms.

In other words, the problem of interest in the *ex post* perspective is to assess the potential impact of a proposed arrangement among patent holders on the functioning of the market in subsequent periods. The objective is to clear pools that will not raise more antitrust concerns than in the absence of pool. By contrast, what is at stake in the *ex ante* perspective is the magnitude of firms' incentives to invest in R&D as enhanced by the looming possibility to participate in the formation of a pool.

We believe that an *ex ante* perspective for analyzing patent pools is important because it takes into account the specific nature of the patents they include. Unlike standard goods, patents exist for their incentive properties. Every manipulation of the value of patents has an impact on these properties and may change substantially the usefulness of patents as a means to encourage R&D. Only a few theoretical papers adopt this perspective. Lerner and Tirole (2007) extend their previous analysis of independent licensing. They offer simple theoretical foundations to the conjecture that pools with independent licensing are preferable to the absence of pools, as they lead to more profits and make users better off. This strengthens the case in favour of pool agreements that allow firms to license their R&D outputs separately. In our analysis, we evacuate the antitrust concerns of all kinds by assuming that potential requirements by the antitrust authority are satisfied. In particular, we rule out substitutable patents, we assume that pool members may license their inventions separately, and that new patents may integrate the pool. We also limit the potential strategic externalities to concentrate on the profits enhancing property of pools. The value of a patent in a pool is supposed not to depend on the size of

non-cooperative process of pool formation.

the pool. Similarly, the value of a patent outside the pool is supposed not to depend on the existence nor on the size of the pool. This allows us to analyze the impact of the formation of a pool, as well as the impact of the date of formation of this pool, on the speed of innovation.

Our analysis is also complementary to a recent paper by Gilbert and Katz (2007), who study formally how alternative reward schemes impact the choice of R&D levels by two firms. We have in common the specification that technologies are invented sequentially, and are more valuable when used together than separately, although they result from independent and uncertain R&D processes. An important difference is that Gilbert and Katz (2007) identify the properties an innovation reward scheme must satisfy to support efficient R&D investment, while we specify the simplest possible reward structure before examining its effect on firms' R&D efforts.⁶ Another difference is that, in our model, pool initiators are not identified *a priori*. Only the pool size is technologically determined, so that firms compete to belong to the subset of early innovators that may form the pool.

The present analysis of patent pooling also bears some resemblance with the standard literature on the *ex ante* study of patenting as exemplified by Reinganum (1989) or Grossman and Shapiro (1987) for instance. In both settings, incentives to innovate are provided by a “prize” in case of success. However, whereas the monopoly power associated to a patent is costly to the regulator, the formation of a patent pool can be beneficial even from a purely *ex post* viewpoint because it facilitates the diffusion of the innovations. This difference has some consequences for welfare analysis.

The paper is organized as follows. Section 2 builds a model of pool formation in the terms of a differential game. Section 3 offers a characterization of the Symmetric Markov Perfect Equilibria

⁶For an empirical analysis of the connection between a firm's decision to participate in a pool and profit sharing schemes, see Layne-Farrar and Lerner (2006).

of the game. Section 4 is an analysis of the impact of the size of the pool on the incentives to invest in R&D and on the speed of innovation. Section 5 compares the equilibrium R&D investment choices with joint profit-maximizing levels and discusses the welfare implications of the pool review process. Section 6 is a conclusion. All proofs can be found in the Appendix.

2 The Patent Race Game

We consider an industry of N symmetric firms. Each of them can obtain exactly one innovation by investing in a specific R&D program. Each innovation is “essential”, in the sense that it has no substitute. When it innovates, a firm is granted a patent of infinite length and thereby can secure a given flow of profits \underline{v} (say, by licensing the innovation), which does not depend on the existence of other patents nor on their inclusion in a pool. Investment in an R&D program is a continuous time profit-maximization decision problem. At each point in time, each firm i can decide independently to exert a non-negative R&D effort x_t^i at some flow cost $c(x_t^i)$, which is such that $c(0) = 0$, $c'(x_t^i) \geq 0$, and $c''(x_t^i) > 0$. We also assume that $c'(0) = 0$ and $c'(+\infty) = +\infty$, for a firm’s optimal choice of effort to be an interior solution of the corresponding program.⁷ Innovation is described by a Poisson process and x_t^i is normalized to be firm i ’s instantaneous probability of success.⁸ Our formulation of the patent race game is thus a particular case of Reinganum (1981).

We denote by $K \leq N$ the minimal number of patents necessary to constitute a pool in a given technological domain. Here K describes firms’ anticipations on the pool formation process when they choose a level of effort in a risky R&D program. In a more complex setting that would model the interaction between firms and antitrust authorities, K would be determined in

⁷The specification that firms incur a variable cost in the R&D technology is as in Lee and Wilde (1980). However, we consider the possibility that firms instantaneously adjust their rate of effort x_t^i .

⁸The Poisson assumption implies that there is no accumulation of knowledge, see Reinganum (1989).

equilibrium.⁹ In the present model, we simply consider K as a non-stochastic parameter which is identical for all firms. In addition, we assume that firms' anticipations are correct, in the sense that the pool is effectively formed when K innovations are available.

Remark that any subset of firms may form a pool that reflects the technological outcome of their respective R&D outputs. Together with the assumption that patents are essential, this implies that the new technology the pool delineates is not pre-determined at $t = 0$. To see this, suppose for an example that $N = 5$ and $K = 3$. In that case, if firms 1 to 3 are the first to innovate and create a pool, the resulting technology (say, an audio and video encoding technology for digital television signals, as in MPEG-2) is not the same as what would be obtained when firms 2 to 4 innovate first to form a pool that supports a distinctive technology (say, an audio and video compression technology for Internet downloads and streaming, as in MPEG-4). The pool will converge over time to a fully integrative technology (all applications for audio and video encoding, compression, and related uses) if the remaining firms eventually succeed in patenting an innovation and join the existing pool. This means that, in this model, firms do not target a particular set of patents that fits a pre-established standard. This is the very possibility of forming a pool, independently of the technological properties of its constitutive patents, that impacts firms' incentives to invest in R&D.

Patent pooling is a cooperative agreement, which establishes the rules according to which pool profits are shared among individual firms. We consider the symmetric case and assume that, once the pool is created, all resulting profits are shared equally among pool initiators. If the marginal contribution of each innovation to the aggregate pool value is constant (i.e., no

⁹Modelling the interaction between the regulator and the firm would require to deal with the rather complex issue of out-of-equilibrium beliefs of firms, i.e. the beliefs hold by the firms about K if they observe an unexpected action of the regulator. To study our problem and obtain the main results, there is no need to introduce such complications.

patent is “better” than others), the equal sharing of profits assumption implies that all patents in the pool generate a flow of profits, we denote by \bar{v} , which does not depend on the pool size.¹⁰ The specification of a simple equal profit-sharing rule facilitates formal developments. It also happens to reflect several important real-world pool cases. An example, that Layne-Farrar and Lerner (2006) qualify as “typical”, is MPEG-2. In this pool, as in a majority of agreements studied in the literature, a numeric proportional rule applies, so that members receive a share of the earnings solely in relation to the *number* of patents they contribute to the pool.¹¹

However, as we shall see, the actualized value of participating in a pool does depend on N and K . The fact that a pool increases the private value of patents (say, because it coordinates the diffusion of a standard) is reflected by the inequality

$$\bar{v} > \underline{v}. \tag{1}$$

As Scotchmer (2004) puts it, “[p]rospective inventors face different rewards if their intellectual property goes into a patent pool than if they license individually” (p. 178). In addition, an important qualitative feature we want to capture is the fact that a patentee would rather be in the set of pool initiators than become an incremental contributor obliged to negotiate its entry in an existing pool. Therefore we assume that the pool initiators have full bargaining power when they deal with potential entrants.¹² As the established pool cannot commit to reward new

¹⁰It is not clear that a more realistic specification would be to have a patent value that increases with pool size. Indeed there might be some threshold effects, so that the marginal contribution of a new patent to the pool value increases until the pool reaches a given size, and decreases afterwards. Our working assumption that the value of patents in the pool is a constant \bar{v} is thus reasonable as it facilitates computations without contradicting any kind of *prima facie* empirical evidence.

¹¹Layne-Farrar and Lerner (2006) analyze nine modern patent pools, namely 1394, AVC, Bluetooth, DVB-T, DVD-1, DVD-2, PRK, MPEG-2, and MPEG-4. They find that five of them (1394, AVC, DVB-T, MPEG-2, MPEG-4) share earnings by applying a numeric proportional rule, whereby members get a share of total earnings that is function of the number of patents they pour in the pool. Three other pools (DVD-1, DVD-2, PRK) use value-added rules, so that shares are defined in proportion to the value of patents. Only one pool (Bluetooth) operates under a royalty-free licensing policy.

¹²The literature on licenses typically considers a monopolistic entity – a laboratory – which sells a patented technology to a set of firms by making take-it-or-leave-it offers. By the same token, in the present case the pool

inventors more than their reservation profits, a firm that contemplates integrating it can only expect a flow of profits \underline{v} from the patent it holds.

These simplifying assumptions are made to keep the model tractable and nevertheless allow us to study the dynamic incentives to perform R&D. To do that, in the following paragraphs we first examine the no-pool benchmark case. Then we establish the actualized value of the patents owned by the pool initiators and by outsiders, before describing a recursive formulation of the race to pool creation.

The No-Pool Benchmark Case Suppose that no patent pool may be formed. Since R&D programs are independent of one another, the environment is non-strategic and we can analyze any firm's decision by studying the optimal pattern of investment for a single representative firm. The maximum value function V of the firm's program depends on the innovation state, which can take values 0 or 1. In case of success, or state 1, the actualized value of a patent is $V(1) = \underline{v}/r$, where r is the common interest rate. In the absence of innovation, or state 0, the actualized value $V(0)$ of the firm's R&D program verifies

$$rV(0) = \max_x [x(V(1) - V(0)) - c(x)]. \quad (2)$$

In the usual terminology, the latter Bellman equation says that the return r on the "asset" $V(0)$ is equal to the expected "capital gains" $x(V(1) - V(0))$ minus the flow of "dividends" $c(x)$. The first-order condition for an optimal level of R&D effort implies that

$$V(0) = \underline{v}/r - c'(\underline{x}), \quad (3)$$

where \underline{x} is solution to

$$\underline{v} - (\underline{x} + r)c'(\underline{x}) + c(\underline{x}) = 0, \quad (4)$$

initiators – considered as a whole – are assumed to benefit from a monopolistic position when they sell outsiders the right to enter the pool.

as obtained by plugging (3) into (2).

The Formation of the Pool The pool is formed by the K first patentees, which thereby gain two kinds of benefits. First, they make a higher return on their own patent, that is \bar{v} , than in the benchmark case. In addition, they extract some rent, that is a flow of profits $\bar{v} - \underline{v}$, from any subsequent innovator interested in entering the pool. If we denote by $V_K(1)$ the actualized value of being entitled with a patent at the date of pool formation, we have

$$V_K(1) = \frac{\bar{v}}{r} + \frac{N - K}{K} \frac{x}{x + r} \frac{\bar{v} - \underline{v}}{r}. \quad (5)$$

The interpretation of the right-hand side of the latter expression is rather simple. The first term is the actualized value of a patent that forms the pool and brings a flow of profits equal to \bar{v} . The second term reflects the fact that each of the potential $N - K$ subsequent entrants contributes to the total pool value by a flow of rent equal to $\bar{v} - \underline{v}$. The total flow is actualized (divided by r), discounted by the “adjusted probability” of success of the R&D programs $\frac{x}{x+r}$, and equally divided among the K pool initiators.¹³

It is easy to check that $V_K(1)$ increases with the optimal level of effort by firms that remain outside the pool, and with their adjusted probability of success, as one would expect. Note that $V_K(1)$ also increases with the number of firms outside the pool. In other words, from the group of pool founders’ viewpoint, the larger the number of potential contributors to the existing pool, the more profitable. It is equivalent to observe that, for a given total number of firms, the value

¹³When the length of one period goes to infinity, the probability of success of the R&D program goes to 1. However, time is costly and from the expected profit expression we can derive an “adjusted probability” of success. Suppose that the stationary investment of the firm is x and the size of the reward in case of success is Π . Recalling that x is the instantaneous probability of success of a Poisson process, the firm’s expected profit is

$$\pi = \int_0^{+\infty} x e^{-xt} e^{-rt} \Pi dt = \frac{x}{x+r} \Pi.$$

In words, everything happens as if by investing x the firm were instantaneously successful with probability $\frac{x}{x+r}$.

of participating in the formation of the pool decreases with the number of founders, as it means sharing the rent extraction process with more parties afterwards. Interestingly, this property has been the object of an empirical test by Layne-Farrar and Lerner (2006) that lends support to the present theoretical formalization.

Eventually, after the formation of the pool, the R&D decisions of all outsiders – the firms which have not patented yet and thus do not participate in the pool – are the same as in the no-pool benchmark case. Each of them invests \underline{x} , that is the solution to (4), and the actualized value of each R&D program is as displayed in (3), that is

$$V_{K+1}(0) = \underline{v}/r - c'(\underline{x}). \quad (6)$$

In other words, (6) is the value of not being among the K first patent holders.

A Recursive Formulation of the Race to the Pool We now concentrate on the period starting at date 0 and finishing with the formation of the pool. This period is analogous to a race in which the prize consists in being among the K first innovators, as this gives access to a portion of the pool value. Equivalently, we choose to describe it hereafter as a series of K successive patent races. The environment is now strategic because each firm's expected return from a patent depends on the achievement of other firms to be among the K first patentees, and thus on their respective investments. This implies that the maximum value of a firm's R&D program is function not only of the innovation state (i.e., to be successful or not), but also of the number of firms which already patented an innovation. As we assume that choices in R&D investments depend only on a current value relevant state, and firms are identical, we restrict attention to Symmetric Markov Perfect Equilibria. Value functions can thus be indexed only by the number of firms that have already patented an innovation or, equivalently, by the rank k

of the patent race in which a firm participates, with $k \leq N$. Formally, by generalizing previous notation, we denote by $V_k^i(0)$ the actualized value of the research program (for firm i) that aims at discovering the k -th innovation (when there exist $k-1$ patentees), and by $V_k^i(1)$ the actualized value of patenting in the k -th race (in which case there are k patentees).

When exactly $k-1$ firms have patented an innovation, $N-k+1$ other firms keep investing in an R&D program. Let us concentrate on one of these firms, we label i , in order to compute its equilibrium strategy x^i . Firm i 's R&D program can either be successful before others and thus lead to an innovation of value $V_k^i(1)$, or fail in patenting the k -th innovation and be valued $V_{k+1}^i(0)$.¹⁴ It follows that the actualized value $V_k^i(0)$ of firm i 's R&D program of rank k verifies

$$rV_k^i(0) = \max_x [x(V_k^i(1) - V_k^i(0)) + X^j(V_{k+1}^i(0) - V_k^i(0)) - c(x)], \quad (7)$$

where X^j is the sum of the instantaneous R&D efforts made by all other participating firms $j \neq i$. In words, the latter displayed expression equates the return on the “asset” $V_k^i(0)$ to the expected “capital gains” minus “dividends”. In contrast to the no-pool benchmark case, here capital gains can take two forms, depending on whether firm i innovates first or another firm j gets ahead. The first-order condition leads to firm i 's optimal effort strategy

$$x_k^i = (c')^{-1} (V_k^i(1) - V_k^i(0)). \quad (8)$$

Now we may use the symmetry assumption to write $V_k^i(1) = V_k(1)$, $V_k^i(0) = V_k(0)$, and consequently $x_k^i = x_k$, for all i , with

$$x_k = (c')^{-1} (V_k(1) - V_k(0)). \quad (9)$$

This leads us to transform (7) into

$$rV_k(0) = x_k c'(x_k) + (N-k)x_k(V_{k+1}(0) - V_k(0)) - c(x_k). \quad (10)$$

¹⁴When a firm participates in some race k and does not succeed in being the first to discover an innovation, it initiates a new R&D program of rank $k+1$. Accordingly, the actualized value of a program that fails to patent the k -th innovation can be denoted by $V_{k+1}^i(0)$. For $k=N$, we adopt the notational convention that $V_{N+1}^i(0) = 0$.

Finally, to characterize completely the value function associated with this game, we must compute the value of patenting in the k -th race, that is $V_k(1)$. This is done by observing that during the $k + 1$ -th race, that is in the period that follows the discovery of the k -th innovation, and before the discovery of another innovation, each of the k patentees receives a flow of profits equal to \underline{v} . In the same period, the event that one of the “remaining” $N - k$ firms succeeds in patenting an innovation can occur with an “adjusted probability” of success of $\frac{(N-k)x_{k+1}}{r+(N-k)x_{k+1}}$, in which case the actualized value of all patents at the issue of the race is equal to the value of innovating at rank $k + 1$, that is $V_{k+1}(1)$. Otherwise, all R&D programs fail with probability $\frac{r}{r+(N-k)x_{k+1}}$, and the actualized value of each existing k patents remains equal to $V_k(1)$. This leads to

$$V_k(1) = \frac{r \left(\frac{\underline{v}}{r}\right) + (N - k) x_{k+1} V_{k+1}(1)}{r + (N - k) x_{k+1}}. \quad (11)$$

In other words, $V_k(1)$ is the value of a lottery, in which a firm can gain $\frac{\underline{v}}{r}$ with probability $\frac{r}{r+(N-k)x_{k+1}}$, or $V_{k+1}(1)$ with probability $\frac{(N-k)x_{k+1}}{r+(N-k)x_{k+1}}$. Monotonicity properties for $V_k(1)$ as a function of k appear in the next section, in which we characterize the solution of the dynamic pool formation program.

3 Equilibrium Pattern of R&D Efforts

In this section we characterize the Symmetric Markov Perfect Equilibria of the K races. The two main properties we obtain in a first proposition are that equilibrium R&D efforts (as made by firms that have not innovated yet) are increasing with time as long as the pool is not formed, and are always greater than the post pool-formation efforts. These basic properties are obtained through a series of lemmas which characterize the solution(s) of the recursive system as formulated above.

Lemma 1 *For all $k \leq K - 1$:*

$$\frac{v}{r} < V_k(1) < V_{k+1}(1).$$

This says that the value of a patent increases as the rank k gets closer to the pool formation rank K . Indeed, the reward accruing to a patentee which participates in the formation of the pool is less discounted in race $k + 1$ than at rank k . Moreover, since owning one of the K first patents gives access to a share of the pool's profits on top of the flow of profits obtained in the no pool situation, we have $V_k(1) > \frac{v}{r}$.

Lemma 2

$$x_K > \underline{x}.$$

This claim becomes intuitive if one observes that, in the K -th patent race, firms have a last chance to participate in the formation of the pool. This perspective encourages them to choose a more aggressive strategy than in the absence of hope to benefit from the pool's profits. Exacerbated competition in this last race induces a higher effort level than in the post pool-formation period. The next lemma establishes the monotonicity of equilibrium investment efforts.

Lemma 3 *The two following properties are equivalent:*

(i) $x_k < x_{k+1}$;

(ii) $\underline{x} < x_k$.

By combining those statements, we can now state the main result of this section.

Proposition 1 For all $k \leq K - 1$:

$$\underline{x} < x_k < x_{k+1}.$$

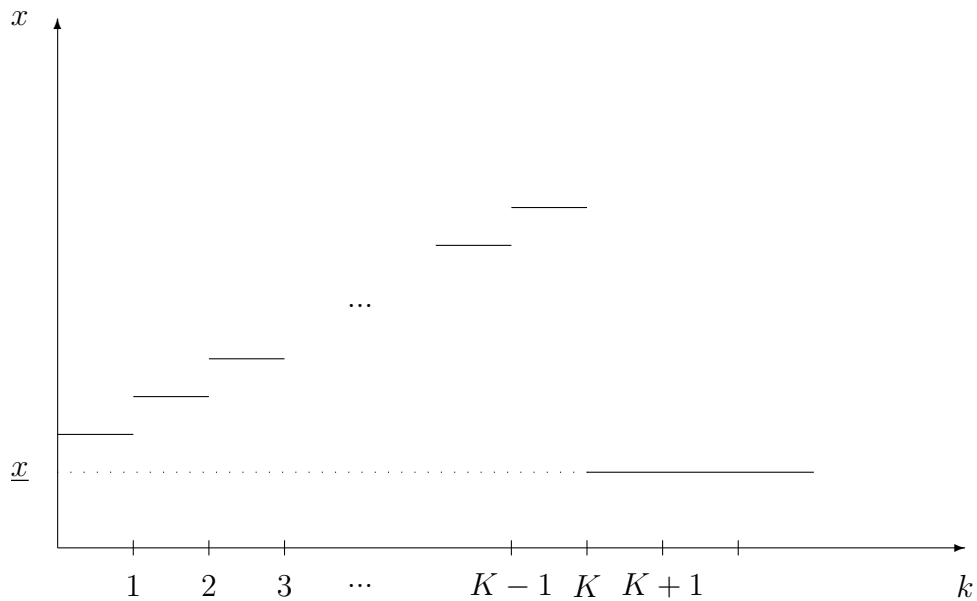


Figure 1: Pattern of Equilibrium R&D Effort Levels.

Figure 1 summarizes the information given by this proposition on the equilibrium R&D effort levels of race participants.

In words, before the formation of the pool, race participants' R&D efforts increase with the rank of the race. After the formation of the pool, outsiders' R&D efforts decrease drastically. Proposition 1 characterizes the impact on R&D activity of the possibility given to firms to form a patent pool of a given size K . Compared to a situation in which no pool can be formed, the speed of innovation is higher before the formation of the pool (i.e., $x_k > \underline{x}$). By increasing

the prospective profits of a patent, the pool acts as an additional reward, and thus enhances incentives to perform R&D.

Another consequence of Proposition 1 is that, as the number of patents gets closer to the pool formation level, successive races become increasingly tough, inducing the participants to raise their level of effort. This characterization is rooted in three distinct effects.

In the k -th patent race, the number of participants is larger than in the $k + 1$ -th race. It is well known that in models *à la* Lee and Wilde, and under some stability conditions, this enhanced competition should increase the equilibrium level of R&D for each firm. However, in our model, this is counterbalanced by the two following effects. Firstly, if a firm wins the k -th race, then it waits longer for the formation of the pool than if it succeeds in patenting the $k + 1$ -th innovation. Time being discounted, the reward is thus smaller in the k -th race. In addition, the failure to innovate is less damaging in the k -th race than in the $k + 1$ -th race. The reason is that, at the lower rank, there is one more race to run before the formation of the pool, and thus one additional chance to be among the K first patentees. The latter two effects dominate the first one (whatever the stability properties of the equilibrium), leading to $x_k < x_{k+1}$.

Remark that we did not address so far the question of the uniqueness of the solution to the recursive system. Actually, it is not clear that the Bellman equation (7) has only one symmetric solution, meaning that the differential game we consider may have several Symmetric Markov Perfect Equilibria. However, Proposition 1 was proved without relying on uniqueness. This means that our results are valid for all possible equilibria. In all equilibrium paths, race participants' R&D efforts increase over time until pool formation.

An empirical implication can be derived from this first result. To see that, consider the number of patent applications as a proxy for R&D intensity, in all industries in which a pool

was formed. These patents may be counted period by period (say, on a monthly basis). By controlling for other R&D incentives, one expects the flow of technologically related patent applications to increase over a time interval that precedes the formation of a pool, and to fall at a lower level afterwards.

We end this section with a lemma that will be useful for subsequent analysis.

Lemma 4 *For all $k \leq K$:*

$$V_k(0) > V_{K+1}(0) = \frac{v}{r} - c'(\underline{x}).$$

In more intuitive terms, this lemma says that the perspective of the formation of a pool increases the potential value of an innovation, and thus also increases the value of all research programs.

4 Size Effects

We already know from Proposition 1 that, before the formation of a pool of given size K , firms' R&D efforts increase with the rank k of the race, with $k \leq K$. In this section we want to analyze the impact of the size of the pool K on the incentives to invest in R&D, and thus on the speed of innovation. Because the perspective of a pool acts as a reward on investment, increasing the pool size from \underline{K} to \bar{K} should *a priori* raise effort levels that aim at patenting in races \underline{K} to \bar{K} , all other things remaining equal. We will see with Proposition 2 that this is not the only impact of an increased pool size, since this also results in a *reduction* of efforts in the \underline{K} first races.

As the proof of the results below make use of the uniqueness of the equilibrium solution to the recursive system, we start by identifying the following mild sufficient condition.

Lemma 5 *If $c''' \geq 0$, there exists only one Symmetric Markov Perfect Equilibrium.*

From now on, we introduce the additional assumption that $c''' \geq 0$.

Because we want to compare the Symmetric Markov Perfect Equilibria we obtain by considering different values of the pool size, we extend the previous notation by explicitly incorporating K in the arguments of the value function and the effort level. We now denote by $V_k(1, K)$ (resp. $V_k(0, K)$) the value associated with a patent (resp. a research program) when exactly k innovations (resp. exactly $k - 1$ innovations) have been patented, and K patents are needed to form a pool. The corresponding effort level of the firms which participate in the k -th patent race is $x_k(K)$.

Lemma 6 *$V_K(1, K)$ is a strictly decreasing function of K .*

This claim is derived directly from equation (5), and results from the fact that the value of participating in the formation of the pool gets lower if the size K increases, since the benefits will be reduced – there will be less potential entrants from which to extract a rent – and will be shared by a larger number of insiders.

Proposition 2 *For two different pool sizes, $\underline{K} < \overline{K}$, the equilibrium patterns of innovative effort levels verify:*

- (i) for $k \leq \underline{K}$, $x_k(\underline{K}) > x_k(\overline{K})$;
- (ii) for $k > \overline{K}$, $x_k(\overline{K}) = x_k(\underline{K}) = \underline{x}$;
- (iii) for $\underline{K} < k \leq \overline{K}$, $x_k(\overline{K}) > \underline{x} = x_k(\underline{K})$;
- (iv) $x_{\underline{K}}(\underline{K}) > x_{\overline{K}}(\overline{K})$.

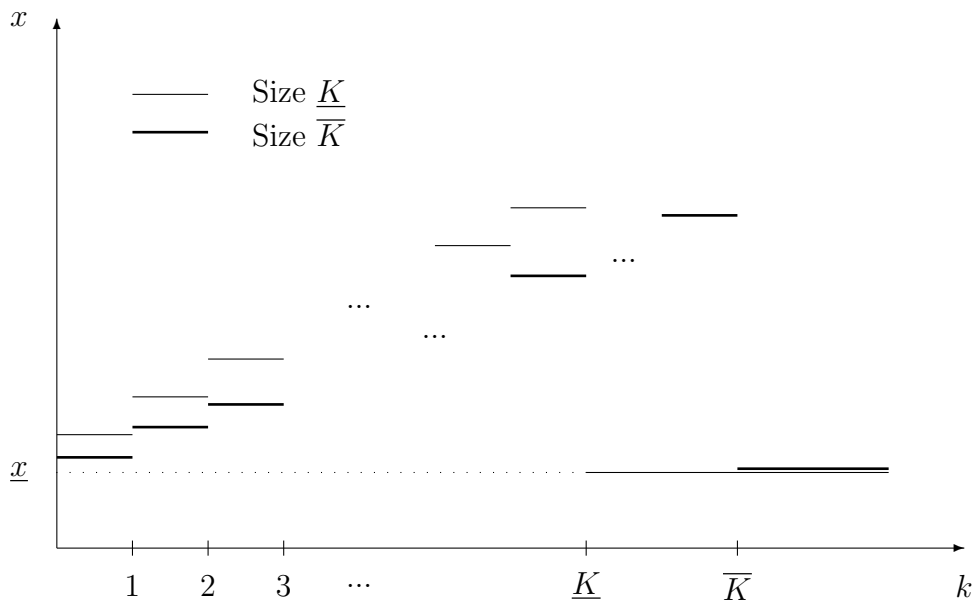


Figure 2: Comparing Two Pool Sizes.

In words, Proposition 2 offers a complete comparison of equilibrium R&D effort patterns as obtained in the small and large pool cases: (i) establishes that race participants' R&D efforts at each rank k in the small size case are higher than in the large size case; (ii) states that the post pool-formation R&D efforts of outsiders fall at the no-pool optimal level \underline{x} for all pool sizes (whereas pool initiators do not invest anymore); eventually, an increase in pool size implies a prolongation of race participants' equilibrium R&D efforts, which according to (iii) are above the no-pool optimal level, and from (iv) are below the final race level of a smaller sized pool. All results given in Proposition 2 are summarized in Figure 2.

An interesting implication of Proposition 2 is that increasing the minimum size of the pool dilutes the incentives to obtain the first patents, and thus decreases the equilibrium R&D efforts

for the discovery of innovations. This important effect is the consequence of three distinct factors. First, patenting in the k -th race is more valuable if K is smaller because the discount applied on the reward is smaller (the waiting time is shorter). Second, the reward itself is higher for a smaller K because the pool's profit pie is not only larger but also divided among a smaller set of firms. Finally, the larger K , the less damaging a failure in the k -th race, since the number of opportunities to belong to the set of pool initiators (that is, the number of subsequent races $K - k$), as faced by an unsuccessful firm, increases with the minimum size of the pool. All three factors contribute to the claim that, although unsuccessful firms have an incentive to increase their R&D effort successively from the first to the K -th race, a reduction in the size of the pool implies higher levels of investment in each race.

5 Pre Pool-Formation Incentives and Welfare Consequences

We can now use the results obtained in the previous sections to point to two distinct sources of distortion. The first one affects private benefits and originates from the pre pool-formation incentives that drive firms' R&D investment decisions. The second one impacts social welfare and is rooted in the prospective orientation of the pool review process, as conducted by the regulator.

Early Overinvestment in Innovation Consider for one moment the overall industry acting as a single player and assume that the latter has control over the rank K and the vector $\mathbf{x} = (x_1, x_2, \dots, x_N)$ of investment levels chosen by all firms, taken as a whole, as if they were cooperating in R&D. This player will maximize the joint profits of the innovative firms. As the effect of the pool formation is to increase the value of innovations (from \underline{v} to \bar{v}), a simple revealed preference argument leads to the conclusion that it cannot be worse off by choosing $K = 1$ than

any other pool size.¹⁵ The profits accruing to the industry for each innovation are then \bar{v}/r and the optimal investment level in each project is \bar{x} , as implicitly defined by the first-order condition $\bar{v} - (\bar{x} + r)c'(\bar{x}) + c(\bar{x}) = 0$. As $c(\cdot)$ is convex and $\bar{v} \geq v$, we have $\bar{x} \geq x$. This investment level is also the one chosen independently by firms involved in an R&D program with expected return \bar{v} , as demonstrated in Section 2. We refer to the choice $K = 1$ and the levels $\bar{\mathbf{x}} = (\bar{x}, \dots, \bar{x})$ as the joint profit-maximizing strategy of the industry. By using this optimal strategy as a benchmark for the assessment of firms' behavior, we reach the conclusion that a pool formation process induces firms to overinvest in the last innovations that are needed to form the pool. This occurs because, in our model, being among the pool initiators is over-rewarded through the extraction of a rent from the firms that innovate after the formation of the pool. In particular, recall that the private value of the K -th innovator is $V(1, K) = \frac{\bar{v}}{r} + \frac{N-K}{K} \frac{x}{x+r} \frac{(\bar{v}-v)}{r}$, which is strictly greater than \bar{v}/r for all $K < N$. Formally, we obtain the following result.

Proposition 3 *For all $K < N$:*

$$x_K(K) > \bar{x}.$$

This says that the final race to the pool exhibits an overinvestment in innovation compared to the joint profit-maximizing R&D level.

It is of interest to link this new proposition to the research stream on multi-stage patent races that concentrates on the cumulative nature of innovation. This literature is mainly concerned by the problem of rewarding the early inventors for opening the way to subsequent improvements, and generally insists on the insufficient incentives given by patent protection. (See Green and

¹⁵The fact that one patent is sufficient to form a pool is one of our modelling assumptions and may be surprising. Actually, one may consider that two (or more) patents are needed to form a pool. Our hypothesis provides a rationale for considering \bar{v} as the benchmark level.

Scotchmer (1995), Chang (1995) or Matutes, Régibeau and Rockett (1996) for representative contributions.) We find that a patent pool can be conceived as a means to increase the rewards to early inventors, so long as it enhances their bargaining power when the time comes to negotiate the inclusion in the pool of new patents by subsequent innovators. The *ex ante* viewpoint thus reveals patent pools as a policy instrument for stimulating early investments in R&D toward the generation of a sequence of complementary patents. This contrasts with the *ex post* antitrust concern that firms may free ride on other pool participants' discoveries, thereby leading to suboptimal R&D investments, when a pool agreement stipulates the cross-licensing of present as well as future patents (see the *Antitrust Guidelines for the Licensing of Intellectual Property* (1995)).

Delayed Formation of the Pool From a social welfare viewpoint, the pool formation process may also entail some form of distortion. To see this point, let us consider a measure of the social welfare $SW = SW(\mathbf{x}, K)$. In equilibrium $\mathbf{x} = \mathbf{x}(K)$, so that the variable K influences the measure of social welfare in two ways, directly through its impact on the social value of an innovation (this comes from the fact that the social value of an innovation may not be the same whether its corresponding patent is in a pool or not), and indirectly through its impact on the speed of the R&D process (i.e., the levels of investment). We now focus on the basic problem of the regulator that consists in encouraging R&D in situations characterized by the following property:

Property 1 $\frac{\partial SW}{\partial x_i} |_{\mathbf{x} \leq (x_1(1), x_2(2), \dots, x_N(N))} > 0$, for any i .

When the latter property is verified, the function SW is assumed to be increasing in x_i , on the domain relevant for our analysis, that is for all R&D investments that are lower than what

is possibly chosen by any firm in equilibrium. As all firms invest \underline{x} at each point in time if no pool may form, and $x_k > \underline{x}$ for all $k \leq K$ otherwise, a direct implication of Property 1 is that the social welfare is higher when the legal context gives firms the opportunity to participate in the formation of a pool.

The analysis of recent cases shows that, in the U.S. context, the decision by competition authorities to clear or not the formation of a pool has been solicited by firms. As Layne-Farra and Lerner (2006) put it, “[r]ecognizing the risk of antitrust intervention has led modern pools to solicit approval from competition authorities *in advance of creating a pool*, as MPEG-2 did” (p. 10, added emphasis). In the present model, this means that K is directly or indirectly fixed by the regulator. From an *ex ante* perspective, the socially optimal choice of K by the regulator is captured by the maximization of $SW(\mathbf{x}(K), K)$. So if the regulator were consulted at date 0 (when $k = 0$) and could commit to a level K , it would choose the level \underline{K} that maximizes $SW(\mathbf{x}(K), K)$. Suppose now that social welfare, as a function of K , is such that there exists a unique maximizer \underline{K} and that, independently of other legal or institutional provisions of all kinds, the review process specifies that the regulator may be consulted by patent holders at any $k \geq 0$. At that point, the regulator can impose conditions (in our model, it can impose a new size $K > k$) for the pool to be eventually cleared. We consider all situations in which the regulator is consulted *after* date 0, that is when $k - 1$ innovations have already been obtained with all firms anticipating the creation of a pool which includes \underline{K} patents, with $1 < k \leq \underline{K}$. Then let the regulator perform a prospective analysis to select a pool size K that maximizes the relevant social welfare function, which in this case writes $SW(x_1(\underline{K}), \dots, x_{k-1}(\underline{K}), x_k(K), \dots, x_N(K), K)$. If social welfare increases in each firm’s R&D investment, this may result in a rise in the minimum pool size, that is in a choice of $\overline{K} > \underline{K}$. In that case, if firms are sufficiently rational to foresee

accurately the formation of the pool, we obtain that \underline{K} will not be anticipated by firms.¹⁶ This reasoning is grounded on the following formal result.

Proposition 4 *If Property 1 is verified, then for all $k < \underline{K} < \overline{K}$:*

$$SW(\mathbf{x}(\overline{K}), \overline{K}) < SW(x_1(\underline{K}), \dots, x_{k-1}(\underline{K}), x_k(\overline{K}), \dots, x_N(\overline{K}), \overline{K}).$$

In words, the latter inequality says that the (expectation taken at date 0 of the) social welfare corresponding to a choice $K = \overline{K}$ is lower than the (expectation taken at date 0 of the) social welfare obtained if firms anticipate $K = \underline{K}$ during the first $k - 1$ races before knowing that $K = \overline{K}$. This claim is a direct consequence of Proposition 2 (recall that $x_k(\overline{K}) < x_k(\underline{K})$ for all $k \leq \underline{K} < \overline{K}$) and Property 1. In intuitive terms, it says that if the acceptance/refusal decision concerning a pool proposal is made on a purely prospective basis at any $k \geq 0$, then it overlooks the pre pool-formation incentive properties that drive firms' choices of R&D investments. As a consequence, even if $SW(\mathbf{x}(\overline{K}), \overline{K}) < SW(\mathbf{x}(\underline{K}), \underline{K})$, it may be the case that, from k onward, \overline{K} eventually turns out to be the regulator's best choice. This in turn restricts from below the range of sizes K that can be anticipated by the pool candidates. As more time is needed to obtain a larger number of patents, this will tend to delay the acceptance decision more than what would be optimal from an *ex ante* perspective. The cleared pool size may actually be larger than the K that maximizes $SW(\mathbf{x}(K), K)$.

The last result is consistent with the standard analysis of patents. In the more common situation of single patent attribution as well, the regulator has fewer motives to award a monopoly

¹⁶Arguably, the detailed treatment of past cases, as offered by the DoJ Business Review Letters, together with the related comments by business consultants and lawyers offer sufficient guidance for firms to anticipate the outcome of a pool review process from the outset (for example, see Morse (2002)).

power to an innovator *ex post* than in the pre innovation period, in which the necessary incentives to perform R&D are taken into account. However, the case of patent pools is more subtle, since some of their virtues (to coordinate the launch of a standard, to clear blocking positions, to reduce transaction costs, to avoid costly infringement litigation), beyond the exercise of monopoly power, can yield benefits to society also *ex post*. Consequently, the regulator will not waver between the options to clear or not to clear the pool, but rather between the options to clear it now or to clear it later.

Our theoretical analysis may help orienting the evolution of public policy, as it echoes real-world observations that raised questions on observed regulatory practices. It is easy to find anecdotal evidence in the business professional literature that a pool review is a timely process, and that law firms are aware of it. The most recent example, in the list of well documented cases, as studied in the law and economics literature, is the 3G pool. Based on a standard defined in 1999, the pool was given clearance in 2002. One reads in a publication by Swidler Berlin Shereff Friedman LLP (2003) that the review process “moved at a glacial pace”, and that “the parties’ interests were harmed by having to wait three years to get a sign-off from the DoJ” (p. 4). This, in the light of our result that a forward-looking regulator, when consulted by pool candidates, is inclined to favour the formation of larger pools than the social optimum, casts doubts on the relevance of a pool review process *before* the pool forms. The time inconsistent behavior of the regulator we point out can be avoided if firms, when interested in founding a pool, simply choose to bypass the review process and rather to capitalize on the lessons from existing pools that received formal approval by competition authorities.

6 Conclusion

Previous contributions to the emerging IO literature on patent pools have followed antitrust practice in adopting the *ex post* perspective for reviewing pool proposals. We have adopted the *ex ante* viewpoint to build a tractable dynamic model of multi-stage innovation, leading to the formation of a pool, and which captures well documented stylized features. The pre pool-formation situation is described as a series of successive patent races leading to a prize, which consists in founding the pool, and thereby in gaining access to a portion of the pool value. After the formation of the pool, late innovators can integrate the pool only by negotiating entry with the pool initiators.

As a result, the possibility of pooling of patents appears to have a positive impact on R&D activity, in the sense that the speed of innovation is higher before the formation of the pool than compared to a situation in which no pool can be formed. More precisely, as the number of patents gets closer to the anticipated pool formation size, it is found that pool candidates raise their level of effort. A firm's investment pattern is thus upward sloping over time before pool formation, and decreases afterwards.

The analysis offers also a complete comparison of equilibrium R&D effort patterns for two different sizes of patent pools. The main insights are as follows: for a given rank in the innovation series leading to the pool, a race participants' R&D investment is higher in the small pool size case than in the large size case; the post pool-formation R&D investment of outsiders falls at the no-pool optimal level for all pool sizes; eventually, an increase in pool size implies a prolongation of race participants' equilibrium R&D investment below the final race level of a smaller sized pool but above the no-pool optimal level. These results imply that an increase in the number of innovations which are required to form a pool dilutes firms' incentives to obtain the first patents,

and thus decreases the equilibrium R&D investments. This sheds light on the positive impact of the possibility of forming a pool on firms' innovative behavior, a feature that was hardly noticed in the literature, and never investigated in a formal setup.

Eventually, our analysis points to two policy implications. The first one is grounded on the characterization of private incentives as faced by firms in the pre pool-formation race. In our model, giving firms the possibility to form a pool stimulates high R&D efforts by early inventors, in comparison with the joint profit-maximizing benchmark. Patent pools can thus be viewed as a lever for stimulating the generation of the first patents in a sequence of innovation programs. This contrasts with the well-known antitrust concern that, in the post pool-formation period, firms may produce too little innovation efforts by free riding on other pool participants' discoveries. The second policy implication is rooted in the prospective nature of the review process as conducted *in advance* of creating a pool. We obtain that a cost-benefit analysis, when performed by a forward-looking regulator, leads to postpone the formation of a pool proposal, and to enlarge its size. As firms may anticipate the bias, this is likely to result in a larger pool than the social optimum. Therefore our theoretical analysis questions the relevance of the solicitation by firms of an approval by competition authorities *before* a pool forms.

As the understanding of the role of patent pools on firms' behavior is of some importance for innovation and competition policy, more effort is needed to investigate their impact on firms' incentives to invest in R&D, resulting innovations, and welfare consequences. To the best of our knowledge, this paper is the first to address these issues in a dynamic setup. Future work could enrich the analysis by introducing changes in the assumptions. For example, one may consider the cases in which the value of patents changes with the size of the pool instead of reaching a high level (i.e., \bar{v} here) that does not depend on the number of patents. Another interesting

variation would be to soften the assumption that pool initiators have full bargaining power, by specifying that new entrants may obtain more in the pool than outside of it (i.e., more than \underline{v}). This would test the robustness of the obtained results to a larger set of specifications.

7 Appendix

Proof of Lemma 1 The proof is immediate since we know from (5) that $V_K(1) > \frac{v}{r}$, and from (11) that $V_k(1)$ is a convex combination of $\frac{v}{r}$ and $V_{k+1}(1)$. \square

Proof of Lemma 2 To prove this lemma, remark first from (4) that the function $\underline{v} - (x+r)c'(x) + c(x)$ takes the value 0 at \underline{x} and is decreasing in x (because c is convex). From (6),(9) and (10) we can write

$$r [V_K(1) - c'(x_K)] = -c(x_K) + x_K c'(x_K) + (N-K)x_K \left[\frac{v}{r} - c'(\underline{x}) - V_K(1) + c'(x_K) \right],$$

which is equivalent to

$$r \left[V_K(1) - \frac{v}{r} \right] + [\underline{v} + c(x_K) - (x_K + r)c'(x_K)] + (N-K)x_K \left[V_K(1) - \frac{v}{r} + c'(\underline{x}) - c'(x_K) \right] = 0.$$

If we suppose that $x_K < \underline{x}$, then all the terms of this sum are (strictly) positive, which is impossible. Hence we must have $x_K > \underline{x}$. \square

Proof of Lemma 3 Let us start by proving that $x_{k+1} < x_k$ implies $x_k < \underline{x}$. By rewriting (10) using (9), we have

$$r [V_k(1) - c'(x_k)] = x_k c'(x_k) + (N-k)x_k [V_{k+1}(1) - c'(x_{k+1}) - V_k(1) + c'(x_k)] - c(x_k). \quad (12)$$

Moreover, we know from (11) that

$$-(N-k)x_{k+1} [V_{k+1}(1) - V_k(1)] + rV_k(1) = \underline{v},$$

and Lemma 1 together with $x_{k+1} < x_k$ gives

$$-(N-k)x_k [V_{k+1}(1) - V_k(1)] + c'(x_k) - c'(x_{k+1}) + rV_k(1) < \underline{v}.$$

We can now rewrite (12) as

$$\underline{v} - (r + x_k)c'(x_k) + c(x_k) > 0,$$

which in turn implies $x_k < \underline{x}$. For the other implication, $x_k < \underline{x}$ implies that $-rc'(x_k) + c(x_k) - x_k c'(x_k) > -\underline{v} = (N - k)x_{k+1}(V_{k+1}(1) - V_k(1)) - rV_k(1)$, thus from (12) we have

$$(N - k)(x_{k+1} - x_k) [V_{k+1}(1) - V_k(1)] + (N - k)x_k [c'(x_{k+1}) - c'(x_k)] < 0.$$

Then Lemma 1 implies that $x_{k+1} < x_k$ must be verified. \square

Proof of Lemma 4 Assume that the weak inequality is verified for $k + 1$, and suppose that $V_k(0) \leq \frac{v}{r} - c'(\underline{x})$. Then from (10) we derive that $\underline{v} - rc'(\underline{x}) \geq -c(x_k) + x_k c'(x_k)$, which is impossible because $x_k > \underline{x}$. To conclude the proof, remark that the weak inequality is verified at $K + 1$. \square

Proof of Lemma 5 Define the function

$$F(x_k) = rV_k(1) - (r + x_k)c'(x_k) + c(x_k) - (N - k)x_k [V_{k+1}(1) - c'(x_{k+1}) - V_k(1) + c'(x_k)].$$

From (11) we obtain $F(0) > 0$, and if $c''' \geq 0$ then $F'' < 0$. Hence there exists only one x_k such that $F(x_k) = 0$ (which corresponds to a solution of the Bellman equation). By iterating, we demonstrate that there exists only one symmetric equilibrium path. \square

Proof of Proposition 2 The points (ii) and (iii) are straightforward. Let us prove point (i). Consider the case $K = \underline{K}$. We know from equation (11) that

$$V_{k+1}(1, \underline{K}) - V_k(1, \underline{K}) = \frac{r(V_k(1, \underline{K}) - \frac{v}{r})}{(N - k)x_{k+1}(\underline{K})},$$

and replacing that into equation (12) gives

$$r [V_k(1, \underline{K}) - c'(x_k(\underline{K}))] = -c(x_k(\underline{K})) + x_k(\underline{K})c'(x_k(\underline{K})) \\ + (N - k)x_k(\underline{K}) \left[\frac{r(V_k(1, \underline{K}) - \frac{v}{r})}{(N - k)x_{k+1}(\underline{K})} - c'(x_{k+1}(\underline{K})) + c'(x_k(\underline{K})) \right].$$

or equivalently

$$F(x_k(\underline{K}), \underline{K}) = r \left(1 - \frac{x_k(\underline{K})}{x_{k+1}(\underline{K})} \right) \left(V_k(1, \underline{K}) - \frac{v}{r} \right) + v + c(x_k(\underline{K})) - x_k(\underline{K})c'(x_k(\underline{K})) \\ - rc'(x_k(\underline{K})) + (N - k)x_k(\underline{K}) [c'(x_{k+1}(\underline{K})) - c'(x_k(\underline{K}))] = 0.$$

Now suppose that at stage $k + 1$ we have $V_{k+1}(1, \underline{K}) > V_{k+1}(1, \bar{K})$ and $x_{k+1}(\underline{K}) > x_{k+1}(\bar{K})$, where $\bar{K} > \underline{K}$. Then from equation (11) we deduce that $V_k(1, \underline{K}) > V_k(1, \bar{K})$ because in the \underline{K} -case, we give more weight to the most valuable point in the convex combination and this most valuable point is more valuable than in the \bar{K} -case. From the definition of F , we also know that $F(x_k(\bar{K}), \bar{K}) = 0$ and $F(0, \bar{K}) > 0$. Next, we can verify that because $x_{k+1}(\underline{K}) > x_{k+1}(\bar{K})$, $x_{k+1}(\underline{K}) > x_k(\underline{K})$ and $V_k(1, \underline{K}) > V_k(1, \bar{K}) > \frac{v}{r}$, we have $F(x_k(\underline{K}), \bar{K}) < 0$. But then, the intermediate value theorem together with the unicity of the Symmetric Markov Perfect Equilibrium gives $x_k(\underline{K}) > x_k(\bar{K})$. The only thing that remains to be proved now is that $V_{\underline{K}}(1, \underline{K}) > V_{\underline{K}}(1, \bar{K})$ and $x_{\underline{K}}(\underline{K}) > x_{\underline{K}}(\bar{K})$. The first point is a direct consequence of Lemma 1 and Lemma 6. For the second point it is useful to come back to the following expression

$$F(x, K) = rV_k(1, K) - (r + x)c'(x) + c(x) - (N - k)x [V_{k+1}(0, K) - V_k(1, K) + c'(x)].$$

The function F defined in this way verifies $F(0, K) > 0$, $F(x_k(\underline{K}), \underline{K}) = F(x_k(\bar{K}), \bar{K}) = 0$. Using the fact that $V_{\underline{K}}(1, \underline{K}) > V_{\underline{K}}(1, \bar{K})$ and $V_{\underline{K}+1}(0, \underline{K}) < V_{\underline{K}+1}(0, \bar{K})$ (which comes from Lemma 4), we can deduce that $F(x_k(\underline{K}), \bar{K}) < 0$. Again, the intermediate value theorem together with the unicity of the Symmetric Markov Perfect Equilibrium gives $x_{\underline{K}}(\underline{K}) > x_{\underline{K}}(\bar{K})$, as expected.

Eventually, to prove (iv), consider the function $F(x, K) = rV_K(1, K) - (r + x)c'(x) + c(x) - (N - K)x[V_{K+1}(0, K) - V_K(1, K) + c'(x)]$. By definition $F(x_K(K), K) = 0$ and we have $F(0, K) > 0$. Consider $\bar{K} > K$, we know from Lemma 6 that $V_K(1, K) > V_{\bar{K}}(1, \bar{K})$. Moreover $V_{K+1}(0, K) = V_{\bar{K}+1}(0, \bar{K})$ and $V_{K+1}(0, K) - V_K(1, K) + c'(x) < 0$ (Lemma 4). Hence we have $F(x_K(K), \bar{K}) < 0$. The intermediate value theorem together with the unicity of the Symmetric Markov Perfect Equilibrium gives $x_{\bar{K}}(\bar{K}) < x_K(K)$. This concludes the proof. \square

Proof of Proposition 3 Suppose $x_K \leq \bar{x}$. We know that $x_K = (c')^{-1}(V_K(1, K) - V_K(0, K))$ and as $(c')^{-1}$ is increasing and $V_K(1, K) \leq \frac{\bar{v}}{r}$, this implies necessarily that $V_K(0, K) \geq \frac{\bar{v}}{r} - c'(\bar{x})$ or $rV_K(0, K) \geq \bar{x}c'(\bar{x}) - c(\bar{x})$. The hypothesis $c'' > 0$ and the starting assumption $x_K \leq \bar{x}$ together give $V_K(0, K) \geq x_K c'(x_K) - c(x_K)$. But this is inconsistent with equation (10) because $V_{K+1}(0, K) < V_K(0, K)$. Thus $x_K > \bar{x}$. \square

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