Trading book and credit risk: how fundamental is the Basel review?

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Contents

1. Basel recommendations on credit risk
2. Default Risk Charge (DRC) in Basel III FRTB
3. Empirical implications
Credit risk in Basel 2.5 (IRC) and RWA variability

- Basel framework: the Risk Weighted Assets (RWA)

$$\text{Minimum Capital Requirement} = X\% \times \text{RWA}$$ (1)

- RWA for credit risk in the trading book: Incremental Risk Charge (IRC)

  BCBS - Basel 2.5 (2009) [1]

  $$\Rightarrow \text{No prescribed model} \ (\text{internal, often multi-factorial model for the default correlation}).$$

- RWA variability
  - High variability of RWA among financial institutions and jurisdictions.
  - Internal models in cause, especially for the IRC calculation.

  BCBS - RCAP Trading Book (2013) [2, 3]
RWA variability: Hypothetical Portfolio Exercises

Source: Second report on RWA in the trading book.
BCBS - Regulatory Consistency Assessment Program (2013) [2]
Basel III FRTB: the Default Risk Charge (DRC)

- Improving the RWA comparability among financial institutions

⇒ Constraints on the modelling choices for internal models

- **Basel III FRTB, RWA for credit risk:** Default Risk Charge (DRC)
  

⇒ Based on a prescribed two-factor model for the default correlation
Portfolio loss

- **One period portfolio loss**

\[ L = \sum_k EAD_k \times LGD_k \times \text{DefaultIndicator}_k \]  

- Exposures (EAD) and Losses Given Default (LGD) assumed constant for simplicity.

⇒ Focus on correlation modelling.

- **Trading book inventories**

  - Exposures may be long (sign +) or short (sign -).
  - CDS or bond exposures.

- **Latent variable model**

  - Default occurs if a latent variable, \( X_k \), lies below a threshold:

\[ \text{DefaultIndicator}_k = 1_{\{X_k \leq \text{threshold}_k\}} \]  

(3)
Prescribed two-factor model

"The Committee has decided to develop a more prescriptive DRC charge in the models-based framework. Banks using the internal model approach to calculate a default risk charge must use a two-factor default simulation model, which the Committee believes will reduce variation in market risk-weighted assets but be sufficiently risk sensitive as compared to multifactor models."

BCBS (2013) [5]

Factor models

\[ X_k = \beta_k Z + \sqrt{1 - \beta_k' \beta_k} \epsilon_k \] (4)

- \( Z \sim N(0, I_d) \): systematic factor.
- \( \epsilon_k \sim N(0, 1) \) : specific risk.
- \( \beta \in \mathbb{R}^{K,J} \): factor loadings.
- threshold \( k = \Phi^{-1}(p_k) \) with \( p_k \) the default probability of the obligor \( k \) and \( \Phi \) the Gaussian cdf.


Latent or observable factors (sectors, regions, . . .)
Prescribed calibration data

"Default correlations must be based on credit spreads or on listed equity prices".
BCBS (2015) [10]

- Consider \( X \in \mathbb{R}^{K \times T} \) the historical sample of centered returns:

  \[
  \begin{align*}
  \text{Sample covariance matrix :} & \quad \Sigma_{Sample} = T^{-1}XX^t \\
  \text{Shrinked covariance matrix :} & \quad \Sigma_{Shrinkage} = \alpha \Sigma_{FactorModel} + (1 - \alpha) \Sigma_{Sample} \\
  \text{Initial correlation matrix :} & \quad C_0 = (\text{diag}(\Sigma))^{-1/2} \Sigma (\text{diag}(\Sigma))^{-1/2}
  \end{align*}
  \]

- Nearest correlation matrix with a two-factor structure

  \[
  \begin{align*}
  \arg \min_{\beta} f_{obj}(\beta) &= \| C(\beta) - C_0 \|_F \\
  \text{subject to } \beta &\in \Omega = \{ \beta \in \mathbb{R}^{K \times 2} | \beta_k^t \beta_k \leq 1, k = 1, \ldots, K \}
  \end{align*}
  \]

  \( \Rightarrow \) Constraint ensures that \( C(\beta) = \beta \beta^t + \text{diag}(Id - \beta \beta^t) \) is positive semi-definite.

- PCA-based method and Spectral projected gradient method

Specific-systematic decomposition of the loss

\[ L(Z, \varepsilon) = \sum_k EAD_k \times LGD_k \times 1 \{ \beta_k Z + \sqrt{1 - \beta_k^2} \varepsilon_k \leq \Phi^{-1}(p_k) \} \]

- **Hoeffding decomposition of the default losses**
  
  VAN DER VAART (2000) [14], ROSEN & SAUNDERS (2010) [9], HOEFFDING (1948) [15].

\[ L(Z, \varepsilon) = E[L] \]

\[ + E[L|Z] - E[L] \]

\[ + E[L|\varepsilon] - E[L] \]

\[ + L(Z, \varepsilon) - E[L|Z] - E[L|\varepsilon] + E[L] \]

\[ \phi_0(L) : \text{Expected Loss} \]

\[ \phi_1(L; Z) : \text{Systematic Loss} \]

\[ \phi_2(L; \varepsilon) : \text{Specific Loss} \]

\[ \phi_{1,2}(L; Z, \varepsilon) : \text{Interaction Loss} \]

- \( \phi_1(L; Z) \) corresponds (up to the expected loss term) to the heterogeneous Large Pool Approximation.
Portfolio risk and contributions

**Portfolio risk**
- Value-at-Risk: \( \text{VaR}_\alpha[L] = \inf \{ l \in \mathbb{R} | \mathbb{P}(L \leq l) \geq \alpha \} \)
- Full allocation property: \( \text{VaR}_\alpha[L = L_1 + L_2] = \mathbb{E}[L_1|L = \text{VaR}_\alpha[L]] + \mathbb{E}[L_2|L = \text{VaR}_\alpha[L]] \)

**Systematic-specific contribution of the portfolio risk**

\[
\text{VaR}_\alpha[L] = \mathbb{E}[\phi_0|L = \text{VaR}_\alpha[L]] + \mathbb{E}[\phi_1(L; Z)|L = \text{VaR}_\alpha[L]] + \mathbb{E}[\phi_2(L; \varepsilon)|L = \text{VaR}_\alpha[L]] + \mathbb{E}[\phi_{1,2}(L; Z, \varepsilon)|L = \text{VaR}_\alpha[L]]
\]

\( C_{\phi_0} : \text{Expected Loss Contribution} \)
\( C_1(L; Z) : \text{Systematic Contribution} \)
\( C_2(L; \varepsilon) : \text{Specific Contribution} \)
\( C_{1,2}(L; Z, \varepsilon) : \text{Interaction Contribution} \)
### Portfolios - Itraxx Europe - Corporates

<table>
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<tr>
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<th>Long only portfolio</th>
<th>Long/short portfolio</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Composition</strong></td>
<td>Long 125 names</td>
<td>Long 27 financial names</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Short 27 non-financial names</td>
</tr>
<tr>
<td><strong>Exposures</strong></td>
<td>Equaly weighted</td>
<td>Equaly weighted</td>
</tr>
<tr>
<td></td>
<td>Total exposure = 1</td>
<td>Total exposure = 0</td>
</tr>
</tbody>
</table>
1-year Default Probabilities

- **1-year Default Probabilities**: Bloomberg Issuer Default Risk Methodology

![Graph showing 1-year Default Probabilities for Financials and Non-Financials](image)
### Unconstrained correlation matrix and $J$-factor model

<table>
<thead>
<tr>
<th>Configuration</th>
<th>Data for estimating $C_0$</th>
<th>Period</th>
<th>Estimation method for $C_0$</th>
<th>Calibration method for the $J$-factor models</th>
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<td>(1) <strong>Equity - P1</strong></td>
<td>Equity returns</td>
<td>1</td>
<td>Sample correlation</td>
<td>PCA and SPG algorithms</td>
</tr>
<tr>
<td>(2) <strong>Equity - P1 Shrinked</strong></td>
<td>Equity returns</td>
<td>1</td>
<td>Shrinkage ($\alpha_{shrink} = 0.32$)</td>
<td>PCA and SPG algorithms</td>
</tr>
<tr>
<td>(3) <strong>Equity - P1 Exogenous Factors</strong></td>
<td>Equity returns</td>
<td>1</td>
<td>Sample correlation</td>
<td>Linear Regression</td>
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<td>(4) <strong>Equity - P2</strong></td>
<td>Equity returns</td>
<td>2</td>
<td>Sample correlation</td>
<td>PCA and SPG algorithms</td>
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<tr>
<td>(5) <strong>Equity - P2 Shrinked</strong></td>
<td>Equity returns</td>
<td>2</td>
<td>Shrinkage ($\alpha_{shrink} = 0.43$)</td>
<td>PCA and SPG algorithms</td>
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<tr>
<td>(6) <strong>Equity - P2 Exogenous Factors</strong></td>
<td>Equity returns</td>
<td>2</td>
<td>Sample correlation</td>
<td>Linear Regression</td>
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<td>(7) <strong>IRBA</strong></td>
<td>-</td>
<td>-</td>
<td>IRBA formula</td>
<td>PCA and SPG algorithms</td>
</tr>
<tr>
<td>(8) <strong>KMV - P2</strong></td>
<td>-</td>
<td>2</td>
<td>GCorr methodology</td>
<td>PCA and SPG algorithms</td>
</tr>
<tr>
<td>(9) <strong>CDS - P2</strong></td>
<td>CDS spreads</td>
<td>2</td>
<td>Sample correlation</td>
<td>PCA and SPG algorithms</td>
</tr>
</tbody>
</table>

Period 1: from 07/01/2008 to 07/01/2009. Period 2: from 09/01/2013 to 09/01/2014.
Correlation matrices - Distributions

(1) Equity - P1
(2) Equity - P1 - Shrinked
(3) Equity - P1 - Exogenous Factors
(4) Equity - P2
(5) Equity - P2 - Shrinked
(6) Equity - P2 - Exogenous Factors
(7) IRBA
(8) KMV - P2
(9) CDS - P2

Legend:
- Unconstrained model
- 1-factor model
- 2-factor model
- J*-factor model
Impacts on the risk - Long portfolio

Configurations: (1) Equity - P1; (2) Equity - P1 - Shrinked; (3) Equity - P1 - Exogenous Factors; (4) Equity - P2; (5) Equity - P2 - Shrinked; (6) Equity - P2 - Exogenous Factors; (7) IRBA; (8) KMV - P2; (9) CDS - P2. $J^*$-factor model is only active for “(1) Equity – P1” and “(4) Equity – P2” configurations.
Impacts on the risk - Long-short portfolio

Conﬁgurations: (1) Equity - P1; (2) Equity - P1 - Shrinked; (3) Equity - P1 - Exogenous Factors; (4) Equity - P2; (5) Equity - P2 - Shrinked; (6) Equity - P2 - Exogenous Factors; (7) IRBA; (8) KMV - P2; (9) CDS - P2. \( J^* \)-factor model is only active for “(1) Equity – P1” and “(4) Equity – P2” conﬁgurations.
Systematic contribution to the risk - Long portfolio

Configurations: (1) Equity - P1; (2) Equity - P1 - Shrinked; (3) Equity - P1 - Exogenous Factors; (4) Equity - P2; (5) Equity - P2 - Shrinked; (6) Equity - P2 - Exogenous Factors; (7) IRBA; (8) KMV - P2; (9) CDS - P2. $J^*$-factor model is only active for “(1) Equity – P1” and “(4) Equity – P2” configurations.
Systematic contribution to the risk - Long-short portfolio

Configurations: 1) Equity - P1; 2) Equity - P1 - Shrinked; 3) Equity - P1 - Exogenous Factors; 4) Equity - P2; 5) Equity - P2 - Shrinked; 6) Equity - P2 - Exogenous Factors; 7) IRBA; 8) KMV - P2; 9) CDS - P2. J*-factor model is only active for “1) Equity – P1” and “4) Equity – P2” configurations.
Conclusions - RWA variability and comparability

- The RWA variability stemming from correlation modelling remains high.
  - It is a challenge regarding model comparability.
  - Two factor constraint is more active in stressed periods (2008)
  - The prescriptions might prove quite useful when dealing with a large number of assets: unconstrained correlation matrix (with small eigenvalues) would ease the building of opportunistic portfolios.

- Other main sources of variability
  - The high confidence level of the regulatory risk measure;
  - Disparities among correlation matrices (type of data and/or the calibration period).

⇒ Small changes in exposures or other parameters may lead to significant changes in the credit VaR, jeopardizing the comparability of RWA.

- The use of Large Pool Approximation is questionable: poor contribution to the VaR
**Bibliography I**


Bibliography II


