Multiple Exercise Minimum Revenue guarantees in Public Private Partnerships: a primal and dual valuation approach

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Abstract

Public Private Partnerships (PPP) contracts bring together the public and the private sectors in order to provide public service and to develop public infrastructure. If implemented appropriately, these contracts may modernize the public procurement and help governmental agencies overcome the budgetary constraints. One of the key success factors of PPP contracts is the appropriate risk sharing between the public and the private sectors. The Minimum Revenue Guarantee (MRG) is a well spread solution for revenue risk mitigation. The government grants the private sector a minimum revenue that ensures the project profitability. The MRG is a real option with the possibility of multiple exercises before the end of the project. In this paper, we propose a methodology for determining the lower and upper bounds value of this guarantee if the choice of the potential exercise dates is made during the operation phase of the project. An illustration has been made through a hypothetical yet realistic project.

Introduction

The development of PPPs has helped governmental entities to fund new projects in order to fulfill the increasing demand for public service provision. The PPP scheme has known a huge success on both developed and developing countries. Its application has made a huge contribution to the development of infrastructure worldwide. The system provides an effective route to mobilize private funds and makes use of the private business skills for developing public infrastructure.

PPP projects are generally conducted under a project finance framework. Under this scheme, a Special Purpose Vehicle (SPV) is in charge of the project funding, construction and management for a certain period of time (the concession duration) afterwards the project is transferred to the public entity. The SPV is funded by both equity and non-recourse debt. The project success depends solely on the future cash flows that it will generate. Therefore the project has to provide sufficient revenue to cover the debt service and to allow investors recouping a reasonable return on their investment. The concession duration may reach several decades. Unpredictability of future demand and the long investment horizon make PPP projects risky. Public entities have therefore to provide enough incentives for private bidders to make the project appealing.

One of the most significant risks in PPPs projects is the revenue risk. It may lead to huge consequences on the project’s company. The cash flows generated from the project may be considerably insufficient to cover the debt service and to generate the expected return to sponsors. In that case, the project is not viable and the public service quality may be highly affected and in some cases shut down. The revenue risk has a direct impact on sponsors and lenders, but there are no financial consequences for the governmental entity budget. In contrast, the political impact may be substantial if the end users are directly affected by a low-quality service or an interrupted service. Imagine for a while the whole water provision system in a city shut down.

The main characteristics of PPP schemes is that the government has to be cooperative and that public decision makers have to be willing to bail out the SPV in order to avoid political consequences and to ensure the continuity of the public service provision—which is put at stake once the decision to conduct the project under a PPP scheme is made. The PPP scheme is actually a relationship specific investment as it was pointed out by (Dong and Chiara, 2010). In other words, the PPP contract is a dynamic cooperative game where the interest of both parties lies in positive interaction with indeed a variable reward for each party. These rewards depend essentially on each party’s bargaining power and her willingness to make concessions.

The bail-out process and the contract re-negotiation lead undoubtedly to high transnational costs. In order to avoid these costs, flexible terms have to be implemented in the initial contract. This helps achieve a better value for money at least by reducing transnational costs and avoiding unnecessary time for re-negotiation and allocating these resources for other public projects. The most known solution for risk mitigation among practitioners and academics is the minimum revenue guarantee (MRG). In this scheme the government grants the SPV a minimum revenue, which is sufficient to maintain the project viability and by consequence the public service provision. This may be seen as a floor option or a put option on the project’s revenue that the governmental entity offers to the

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operation phase. Let $K$ be the guaranteed revenue and $R_t$ the SPV revenue at year $t$. The government guarantees the SPV the shortfall amount $\max(0, K - R_t)$. This contract is akin to a financial option. Let $M$ be the maximal number of exercise dates. $M$ would be the number of auditing during the project operation phase. We assume that the guarantee is only redeemable at $n \leq M$ exercise dates during the operation phase. There are two alternatives to structure the revenue guarantee:

- A European guarantee contract: the SPV fixes its $n$ exercise dates before the operation phase. This contract is akin to a Portfolio of European options, maturing at each chosen exercise date,
- A Bermudan guarantee contract: the SPV has the freedom to select its $n$ exercise dates during the operation phase. This contract is similar to a Multiple Bermudan Exercise option (swing option).

The importance of revenue guarantee contract and even its optimality in the sense of (Engel et al., 2013) should not decrease the vigilance of the public sector and should not lead to non-reasonable practices. The MRG is a risk mitigation contract. It brings with it the responsibility for all future liabilities and in some cases it may be a huge fiscal and financial burden for taxpayers. This financial commitment is often neglected by some governments who do not explicitly account for the contingent liabilities (Irwin, 2007).

The use of proper tools to assess the value of the MRG is crucial in the sense that it allows both parties reaching "win-win" agreements. This leads to the definition of an adequate guarantee structure, and leads to ensure a minimal project’s viability omitting all chances of excess reward for the private sector. It allows moreover proper accounting and provision for the guarantees.

Pricing MRG had been an active subject of research during the last decade. Lattices method and Monte Carlo simulation were both used to determine the real-option value of the guarantee. MRG was in general seen as a European portfolio of option maturing at fixed dates (Irwin, 2007; Dailami et al., 1999; Brandao and Saraiva, 2008; Ashuri et al., 2011; Ho and Liu, 2002). This approach is quite limiting and contradicting in some extent to the motivation behind the use of revenue-guarantees in PPPs which is essentially adding flexibility to the contract. The European option is very "static" and does not allow the early exercise feature. Moreover, if the risk covers all the concession period, the guarantee value may be too expensive and may increase potentially the project reward.

An alternative approach is to structure these guarantees as Multiple-Exercise Bermudan options. This approach was initiated by (Chiara et al., 2007) who presented a methodology for pricing these options. Their approach was based on The Least squares Monte Carlo algorithm developed by Longstaff and Schwartz (2001). However, there is no unbiased value for this option because of the early exercise feature. The Longstaff and Schwartz (2001) methodology aims to approximate the optimal exercise strategy and provides therefore a lower bound for the option value. Our work proposes to complete this approach. We derive a methodology to determine an upper bound value for the guarantee contract by the means of the dual pricing technique introduced in (Meinshausen and Hambly, 2004). This allows to construct a reliable and a tight confidence interval for the real option value. We investigate furthermore the impact of the number of exercise rights on the project viability.

The remaining of this development is structured as follows: section 1 introduces the minimum revenue guarantee contract. Section 2 summarizes the pricing techniques used in this article. Section 3 illustrates our model via a hypothetical case study. Conclusions and potential improvements are presented in section 4.

1 The minimum revenue guarantee contract

We consider a revenue guarantee contract between a governmental entity and the special purpose vehicle. In this contract, the governmental agency grants the SPV the revenue shortfall occurring at certain years during the operation phase. Let $K$ be the guaranteed revenue and $R_t$ the SPV revenue at year $t$. The government guarantees the SPV the shortfall amount $\max(0, K - R_t)$. This contract is akin to a financial option. Let $M$ be the maximal number of exercise dates. $M$ would be the number of auditing during the project operation phase. We assume that the guarantee is only redeemable at $n \leq M$ exercise dates during the operation phase. There are two alternatives to structure the revenue guarantee:

- A European guarantee contract: the SPV fixes its $n$ exercise dates before the operation phase. This contract is similar to a Portfolio of European options, maturing at each chosen exercise date,
- A Bermudan guarantee contract: the SPV has the freedom to select its $n$ exercise dates during the operation phase. This contract is similar to a Multiple Bermudan Exercise option (swing option).
The main difference between the two alternatives is the decision timing. While in the European guarantee, the decision is taken before the beginning of the operation phase, the decision under the Bermudan contract is made during the operation phase. The Bermudan contract adds more flexibility and allows better hedging for the revenue risk since the SPV can take advantage from the information revealed over time. We assume that the "ideal" MRG contract has to fulfill at least two requirements:

1. ability to ensure the project viability,
2. affordability for the public sector and reduction of the exposure to revenue risk (low exercise rights).

Although the European contract is more affordable to the public sector, it is outperformed by the Bermudan contract. In fact, the Bermudan contract ensures better viability because of the increased flexibility. It can moreover be structured with a small exercise rights which reduces the exposure to the revenue risk. Figure 1 shows that:

- At equal exercise rights, the Bermudan contract offers higher rewards to the private partner.
- At equal cost to the governmental agency, the Bermudan contract offers almost the same reward to the private sector with less exercise rights.

![Figure 1: Comparison between the European and the Bermudan contracts](image)

In the remaining of this development, we assume that a minimum revenue guarantee contract may be decomposed into a floor component and a cap component. The floor component protects the SPV against downside demand and the cap component allows the government to share the profit if the project turns out to be more profitable than projected. The floor component is fully defined given:

- $T_f$: the contract duration,
- $N_f$: number of exercise rights,
- $\Delta t_f$: the fraction of time in years separating two possible exercise dates. It corresponds to the auditing interval of the SPV which may be a trimester, a semester or a year,
- $K_f^t$: is the guaranteed revenue at time $t$.

For the cap component we adopt the same notation with a sub-index $c$. For instance $T_c$ will be the contract duration for the cap component.

We denote by $R_f(t)$ the payoff of the floor component and by $R_c(t)$ the payoff of the cap component and one has:

\[
R_f(t) = (K_f^t - R_f(t))^+, \\
R_c(t) = (R_c(t) - K_c^t)^+,
\]

where $(x)^+ = \max(x, 0)$. The project revenue with the MRG contract $R^*_t$ is given by:

\[
R^*_t = R_f(t) + R_c(t) - R_f(t),
\]

Pricing both component can be made separately since there is no interaction between them. Figure 2 illustrates the MRG contract structure.
2 Multiple exercise Bermudan options pricing by simulation

The pricing of American options by simulation was very challenging until the late nineties. The stochastic dynamic program (SDP) introduced by (Cox et al., 1979) requires the determination of several conditional expectations which is a very hard task to handle using Monte Carlo techniques. The introduction of regression techniques by (Carriere, 1996; Tsitsiklis and Van Roy, 2001; Longstaff and Schwartz, 2001) alleviated the computational burden arising from the conditional expectations evaluation. Such approaches are often known as the “Primal” problem, they aim to approximate the optimal exercise strategy. Another breakthrough to the valuation of American options arising from the conditional expectations evaluation. This section summarizes the primal and dual pricing problems which are used to derive the lower and upper value of the MRG.

2.1 The Primal pricing problem

We consider a discrete time Markov decision processes \( \{X_t\} \) that lives in a subset \( E \) of \( \mathbb{R}^d \) in a filtered probability space \( \{\Omega, \mathcal{F}, \mathbb{P}, \mathbb{Q}\} \). The valuation problem is defined over a finite lifetime \( T \). \( E \) denotes expectation with respect to \( \mathbb{P} \) and \( E_t = E(\cdot|\mathcal{F}_t) \) denotes the conditional expectation at time \( t \). We denote by \( n \in \mathbb{N} \) the number of available exercise rights and we define a payoff function \( Z : [0, \ldots, T] \times E \rightarrow \mathbb{R} \) that measures the reward earned by using an exercise opportunity. We assume that \( E(\max_{t=0, \ldots, T} Z_t) < \infty \) and that \( Z \) is independent of \( n \). We introduce a bank account \( B_t, t \in [0, \ldots, T] \) defined by \( dB_t = r_t B_t dt \) where \( r_t \) is the interest rate at time \( t \). We will take a constant interest rate and though \( B_t = e^{rt} \) and \( B_0 = 1 \). We call a set of stopping times \( \{\tau_n, \tau_{n-1}, \ldots, \tau_1\} \) with \( \tau_n < \tau_{n-1} < \ldots < \tau_1 \) a policy \( \pi \). We define the set of admissible policies by:

\[
\Pi_t(n) = \left\{ (\tau_1, \ldots, \tau_n) \mid \forall k = 1, \ldots, l \leq n : \tau_k < \tau_{k-1} < \ldots < \tau_1, \tau_k \in \{t, \ldots, T\}, \forall k = l + 1, l + 2, \ldots, n : \tau_k := \infty. \right\} \tag{3}
\]

We set \( Z_{\infty} := 0 \). The optimal sequence of stopping times that maximizes the option value over the remaining exercise period \([t, \ldots, T]\) given the current state \( X_t = x \) is denoted \( \pi^*_t = (\tau^*_n(t, n), \ldots, \tau^*_1(t, n)) \in \Pi_t(n) \) and is solution of:

\[
V^*_t(x, n) = \sup_{\pi_t(n) \in \Pi_t(n)} E_t \left[ \sum_{k=1}^{n} Z_{\tau_k}(t, n) \frac{B_{t+k}}{B_{t+k}} X_t = x \right]. \tag{4}
\]

The function \( V^*_t(x, n) \) is the value of the option issued at time \( t \) with \( n \) remaining exercises up to \( T \). The value function can be determined via the Bellman equation:

\[
\begin{align*}
V^*_t(x, n) &= Z_t(x), \\
V^*_t(x, n) &= \max \left\{ Z_t(x) + E_t \left[ \frac{B_t}{B_{t+1}} V^*_t(X_{t+1}, n-1) | X_t = x \right], E_t \left[ \frac{B_t}{B_{t+1}} V^*_t(X_{t+1}, n) | X_t = x \right] \right\}.
\end{align*} \tag{5}
\]
The option holder at time $t$ has the choice between two strategies: exercise the option and hold an option with $n - 1$ remaining exercise possibilities or continue with his option of $n$ remaining rights. This choice is iterated until all rights vanish or the maturity date is reached. The Bellman equation is then iterative by construction in the sense that determining the option price with $n$ exercise rights requires the knowledge of the prices of all the options with $1$ to $n - 1$ exercise rights. We define the continuation value $Q^r_{t+1}(x, n)$ by:

$$Q^r_{t+1}(x, n) = E_t \left[ \frac{B_t}{B_{t+1}} V^r_{t+1}(X_{t+1}, n) \bigg| X_t = x \right],$$

(6)

(Longstaff and Schwartz, 2001) suggested to regress the continuation value into the state space at time $t$ which gives an approximation value $\tilde{Q}^r_{t+1}(X_t, n)$ defined by:

$$\tilde{Q}^r_{t+1} = \sum_{r=1}^{R} \alpha_{r,n,t} \Psi_{r,n}(X_t),$$

(7)

where $\{\Psi_{r,n}(X_t)\}_{r=1}^{R}$ is a set of $R$ basis functions of $E$. The coefficients $\{\alpha_{r,n,t}\}_{r=0}^{R}$ are obtained using least squares regression:

$$\{\alpha_{r,n,t}\}_{r=0}^{R} = \arg \min_{\{\alpha_{r,n,t}\}_{r=0}^{R}} \sum_{h=1}^{N} \left| Q^r_{t+1,h} - \sum_{r=1}^{R} \alpha_{r,n,t} \Psi_{r,n}(X_{t,h}) \right|^2.$$  

(8)

Once the continuation value is approximated, we use it to derive the option value using the Bellman equation:

$$\begin{cases} V_T(x, n) = Z_T(x), \\ V_t(x, n) = \max \left\{ Z_t(x) + \tilde{Q}^r_{t+1}(x, n - 1), \tilde{Q}^r_{t+1}(x, n) \right\}. \end{cases}$$

(9)

### 2.2 The upper bound for multiple-exercise Bermudan option

For the sake of clarity, we denote by $V^\ast_L$ the lower bound and by $V^\ast_U$ the upper bound of the option price. We introduce the marginal value $\Delta V^\ast_U(x, n)$ which measures the value of an additional exercise right:

$$\Delta V^\ast_U(x, n) = V^\ast_U(x, n) - V^\ast_U(x, n - 1).$$

(10)

(Meinshausen and Hambly, 2004) have proved that:

$$\Delta V^\ast_U(x, n) = \inf_{\pi_{n-1}(0, n)} \inf_{M_t \in \mathbb{M}_n} \left\{ E_0 \left[ \max_{t \in \mathcal{T} \cap \{\tau_{n-1}, ..., n\}} (Z_t - M_t(n)) \right] \right\},$$

(11)

where $\mathbb{M}_n$ is the set of all martingales with $M_0 = 0$ and $\mathcal{T} = \{0, ..., T\}$. The martingales can be constructed using an iterative approach:

$$\begin{cases} M_t(n) = M_{t-1}(n) + \Delta V^\ast_U(X_t, k + 1) - \mathbb{E} [\Delta V^\ast_U(X_t, k)] | X_{t-1} = k + 1, \\ M_t(n) = M_{t-1}(n) + \Delta V^\ast_U(X_t, k + 1) - \Delta V^\ast_U(X_t, k) \end{cases}$$

(12)

where $k$ is the largest natural number with $t \leq \tau_k$ and $0 \leq k < n$. It is defined over the stopping times of the option with $n - 1$ rights. The approximation of the expectation in the case $I_{t-1}(X_{t-1}) = 1$ requires to map all the remaining times until the maturity date and is very time consuming. (Andersen and Broadie, 2004; Meinshausen and Hambly, 2004) suggested to generate only one time step ahead from $X_{t-1}$ via a nested Monte Carlo simulation and to compute the expectation value. Once the upper and lower bounds are computed, one can construct a confidence interval at $(1 - \alpha)\%$ of the option price:

$$\left[ V_0(n) - \beta \frac{\sigma_n}{\sqrt{N_t}}, \quad V_0(n) + \beta \frac{\sigma_n}{\sqrt{N_u}} \right],$$

(13)

where $\sigma_n$ (resp. $\sigma_u$) is the volatility of the lower (resp. upper) bound estimation, $N_t$ (resp. $N_u$) is the number of simulation used for the computation of the lower (resp. upper) bound and $\beta = \phi^{-1}(1 - \alpha/2)$ with $\phi^{-1}$ is the inverse cumulative distribution function of the standard normal distribution.
3 Numerical example

3.1 MRG pricing

We consider here the pricing of a MRG using both the primal and dual approach, the confidence interval for the option price is then computed. For this we assume that the revenue process is a standard Geometric Brownian Motion:

\[ dR_t = \mu R_t dt + \sigma R_t dW_t, \]

where \( \mu \) is the revenue increment in \( dt \), \( \sigma \) is the revenue volatility and \( W_t \) is a Brownian motion. \( \sigma \) can be derived from historical data in the case of Brownfield project or from the local GDP variation in the case of Greenfield project.

Here we consider only the pricing of the floor component, a similar approach can be used for the cap component.

For the numerical illustration, we consider the set of parameters:

\[ \mu = 0.01, \quad \sigma = 0.07, \quad R_0 = 20. \]

The minimum revenue guaranteed is given by:

\[ \begin{cases} K_0 = 19, \\ K_{t+1} = 1.02 K_t. \end{cases} \]

For the backward step we use 10000 Monte Carlo simulation, for the forward step we use 10000 Monte Carlo simulation and for the upper loop in the dual approach we consider 20 Monte Carlo simulation and for the nested loop we take 50 Monte Carlo simulation. We take a discounting rate \( r = 10\% \). For regression, we consider the first 7 first Laguerre polynomials defined by:

\[
\begin{align*}
L_0(X) &= e^{\frac{-X^2}{2}}, \\
L_1(X) &= e^{\frac{-X}{2}} (1 - X), \\
L_2(X) &= e^{\frac{-X}{2}} \left(1 - 2X + \frac{X^2}{2}\right), \\
L_n(X) &= e^{\frac{-X}{2}} \frac{d^n}{dx^n}(X^n e^{-X}).
\end{align*}
\] (14)

Table 1 reports our results figure 3 shows the lower and upper value of the option and figure 4 shows the sensitivity of the option price towards \( \sigma, \mu, K \) and the discount rate \( r \).

![Figure 3: Lower and upper bounds for the option value](image-url)
<table>
<thead>
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<th>$V_0(n)$</th>
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<tr>
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<td>-</td>
<td>-</td>
<td>-</td>
<td>[22.800 , 23.748]</td>
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</table>

Table 1: Primal and Dual pricing of Multiple exercise bermudan-style MRG
Figure 4: sensitivity analysis of the option price: (a) sensitivity towards revenue volatility; (b) sensitivity towards revenue increment; (c) sensitivity towards the minimum guaranteed revenue; (d) sensitivity towards the discount rate.
3.2 Impact on the project metrics

In this part we investigate the impact of government guarantees on the project’s financial viability. For this we define three financial indicators:

- \( NPV^s \) : the Net Present value of the project’s sponsors, we use the CAPM to derive the discount rate \( r = 10\% \),
- \( NPV^G \) : the Net Present value of the governmental entity, cash flows are discounted at the risk free rate \( r_g = 4\% \),
- \( \min_2 DSCR \) : the minimal Debt Service Coverage Ratio of the SPV over the operation phase.

We allow convertible debt in the capital structure of the project and we assume that it is detained by the project’s sponsors. And we establish the following exercise hierarchy: the exercise decision for MRG is made first then the conversion decision follows. For a detailed description of cash flows modeling in PPPs please refer to (Zhang, 2005) and for the mezzanine mechanisms to (Dong et al., 2011).

The project \( NPV \) distribution is given in figure 5. Figures 6, 7 and 8 show the project’s financial indicators distributions for some illustrative guarantees. Figure 9 measures the effect of the MRG on the project’s expected financial indicators and on the probability of a negative \( NPV \). The jump seen in the \( NPV_s \) figure is due to the conversion of the mezzanine debt.

One can see clearly the effect of MRG on hedging the sponsor’s risk and increasing the project bankability which is measured by the \( \min_2 DSCR \). Conversely the government risk increases considerably.

![Figure 5: (a): \( NPV^s \) distribution; (b): project revenue](image)

![Figure 6: impact of number of exercise rights on the sponsors : (a) PDF of \( NPV^s \); (b) CDF of \( NPV^s \)](image)
Figure 7: impact of number of exercise rights on the governmental entity: (a) PDF of $NPV^G$; (b) CDF of $NPV^G$

Figure 8: impact of number of exercise rights on the project bankability: (a) PDF of $\min_t DSCR$; (b) CDF of $\min_t DSCR$
Figure 9: project’s expected metrics function of the number of exercise rights: (a) $E(NPV^s)$; (b) $E(NPV^G)$; (c) $E(\text{min} \, DSCR)$; (d) $P(NPV^s < 0)$. 
3.3 Subsidy Vs Guarantees

In this paragraph, we compare direct subsidy to revenue guarantees. For this we suppose that the government is able to give a direct subsidy to the SPV in a manner that reduces its conventional debt while covering all the expense during the construction phase. In other words, the direct subsidy changes the structure of the project’s capital. The level of mezzanine debt does not change. We define the Value at Risk ($VaR$) at level $\alpha$ by:

$$VaR_\alpha = F^{-1}(\alpha),$$

where $F^{-1}$ is the inverse cumulative distribution function of the observed parameter (e.g $NPV^s$). Figure 10 reports our results. One can conclude that in the absence of mezzanine debt, subsidy and minimum revenue guarantees are similar. However if mezzanine debt is considered in the project capital structure and if the guarantee level is sufficient enough to active the mezzanine exercise boundary then MRG outperforms direct subsidy given a fixed level of governmental cost.

For this project, one can see that a level of $(n_f = 15, n_c = 30)$ satisfies both parties.

4 Conclusion

Minimum revenue guarantees are effective for risk sharing between the public and the private sectors in Public Private Partnerships contract (PPPs). These guarantees can be structured as a European contract where the SPV has to fix its exercise dates prior to the operation phase of the project or as a Bermudan contract where the SPV is left the freedom to chose its exercises rights during the operation phase. Bermudan contract are better for both parties involved in PPP contracts because they add more flexibility to the project and allow better risk sharing.

The determination of the MRG contract value is crucial for both parties because it permits to reach “win-win” agreements and to measure the project financial viability. Pricing European MRG is straightforward however, there is no unbiased value for the Bermudan contract due to the early exercise feature. This article presents a methodology for the determination of tight confidence interval for the option value using both the primal (Longstaff and Schwartz, 2001) and the dual pricing techniques (Meinshausen and Hambly, 2004). This techniques were illustrated using a numerical case study. We have studied furthermore, the impact of the Bermudan MRG contract on the project financial viability. It is worth noting, that the quality of the upper bound value is very dependent of the quality of the lower bound which depends on its turn on the quality of the basis functions and on the number of simulations.

The upper bound for the MRG contract can be determined differently using the algorithm developed by (Schoenmakers, 2012) where he derives an other formulation for the dual problem. In (Meinshausen and Hambly, 2004), the optimal martingale is defined over a set of martingales and over a set of stopping times. In (Schoenmakers,
the dual problem is only defined over a set of martingales. His formulation is a natural extension for the dual problem introduced by (Rogers, 2002; Haugh and Kogan, 2004) for American options. Numerical experiments should be therefore conducted to determine which technique gives tighter values for the MRG contract. The MRG contract can include a refraction period, which separates two consecutive exercise dates. This may lead to a better management for the public budgetary constraints. For the valuation of this contract, the methodology presented by (Bender, 2011) can be used. The option value is defined as an infimum over a set of martingales and a set of predictable processes of bounded variation.

References


