The Whys of the LOIS: Credit Skew and Funding Spread Volatility

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Most interest-rate derivatives have LIBOR-indexed cash-flows (LIBOR fixings)

**What is LIBOR?**

- LIBOR stands for London InterBank Offered Rate. It is produced for 10 currencies with 15 maturities quoted for each, ranging from overnight to 12 Months producing 150 rates each business day. LIBOR is computed as a trimmed average of the interbank borrowing rates assembled from the LIBOR contributing banks.

- More precisely, every contributing bank has to submit an answer to the following question: "At what rate could you borrow funds, were you to do so by asking for and then accepting inter-bank offers in a reasonable market size just prior to 11 am?"
### Barclays Bank plc
- **bbalibor Rate**

<table>
<thead>
<tr>
<th>Bank</th>
<th>Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Barclays Bank plc</strong></td>
<td>2.15</td>
</tr>
<tr>
<td><strong>Bank of Tokyo-Mitsubishi-UFJ-Ltd</strong></td>
<td>2.15</td>
</tr>
<tr>
<td><strong>HSBC</strong></td>
<td>2.12</td>
</tr>
<tr>
<td><strong>Royal Bank of Scotland Group</strong></td>
<td>2.11</td>
</tr>
<tr>
<td><strong>UBS AG</strong></td>
<td>2.105</td>
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<tr>
<td><strong>Abbey National</strong></td>
<td>2.1</td>
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<tr>
<td><strong>Bank of America</strong></td>
<td>2.1</td>
</tr>
<tr>
<td><strong>Citibank NA</strong></td>
<td>2.1</td>
</tr>
<tr>
<td><strong>Mizuho Corporate Bank</strong></td>
<td>2.1</td>
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<tr>
<td><strong>Rabobank</strong></td>
<td>2.1</td>
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<tr>
<td><strong>Royal Bank of Canada</strong></td>
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<tr>
<td><strong>WestLB AG</strong></td>
<td>2.1</td>
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<tr>
<td><strong>BNP-Paribas</strong></td>
<td>2.05</td>
</tr>
<tr>
<td><strong>Lloyds Banking Group</strong></td>
<td>2</td>
</tr>
<tr>
<td><strong>Deutsche Bank AG</strong></td>
<td>1.95</td>
</tr>
<tr>
<td><strong>JP Morgan-Chase</strong></td>
<td>1.95</td>
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</tbody>
</table>

**bbalibor Rate** \[= 2.10063 \]
In most currencies there is also an interbank market of overnight loans, at a rate referred to OIS (spot) rate henceforth

- In some currencies the OIS rate (like the EONIA rate for the euro) can be viewed as a short-tenor limit of LIBOR
- In others (like US dollar) this view is simplistic since the panel of the LIBOR and of the OIS rate is not the same, and the OIS rate reflects actual transaction rates (as opposed to a purely collected LIBOR)
Divergence Euribor ("L") / EONIA-swap ("R") rates

**Left:** Sudden divergence between the 3m Euribor and the 3m EONIA-swap rate that occurred on Aug 6 2007

**Right:** Term structure of Euribor vs EONIA-swap rates, Aug 14 2008
- **Drying-up of the interbank market**
- **Segmentation** between LIBOR markets of various tenors since the crises
  - LIBOR-OIS swap spreads, basis swap spreads
  - OIS (overnight) market versus LIBOR 3m market, LIBOR 6m market,..

### Credit risk and liquidity risk fundamentals of these spreads
- Deterioration of the average credit quality of the panelists of a LIBOR during the length of the tenor
  - Credit spread skew constant component


- **MORE OPTIONALITY WHEN LENDING SHORTER**
  - Volatility of the cost-of-capital (including volatility of credit spreads) \( \sqrt{T/2} \)-component

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Some other references


Outline

1. Equilibrium Model
2. LOIS Formula
\[ D_t \] Time-\( t \) short term debt of the bank
\[ \rho_t(D) \] Funding rate of a lending bank with short term debt \( D \)

- A complex output possibly impacted by, depending on the treasury management of the bank, beyond the level of the short term debt \( D \):
  - the O/N \( r_t \)
  - the bank’s CDS spread
  - other macro-economic global variables

\[ \alpha_t = \rho_t(D_t) \] Interest rate charged to the bank “on the last euro that it borrowed” for funding its loan

- “current“ refinancing or funding rate of the bank

\[ \rho_t(D_t + x) \] Interest rate charged to the bank on the last euro of its refinancing debt \( \in(D_t + x) \)

- \( x \) Putative extra amount of refinancing capital borrowed by the bank

OIS or Libor
- **Liquidity issue** $\rho_t(D)$ increasing in $D$ (or $\rho_t(D_t + x)$ increasing in $x$)
  - “the next euro borrowed costs more than the previous one”
- Marginal cost $\rho_t(D_t + x)$ integrated over $x$ from 0 to the cumulative amount $N \rightarrow$ global refinancing cost convex in $N$ (as the integral of an increasing function)
  - **Optionality feature** of the funding liquidity issue
Lending OIS

\[ TU((r_t); (N_t)) = \mathbb{E} \left( \int_0^T N_t r_t dt - \int_0^T \int_0^{N_t} \rho_t(D_t + x) dx dt \right) \leftarrow \max(N_t) \quad (1) \]

- \( N_t \) Amount of notional that the bank is willing to lend at the O/N rate \( r_t \) between \( t \) and \( t + dt \)

Lending Libor

\[ TV(L; N) = \mathbb{E} \left( NL(T \wedge \tau) - \int_0^{T \wedge \tau} \int_0^{N} \rho(D_t + x) dx dt - 1_{\tau < T} N \right) \leftarrow \max N \]

- \( N \) Amount of notional that the bank is willing to lend at the Libor rate \( L \) over the period \([0, T]\)

- \( \tau \) stylized default time of the borrower reflecting the deterioration of the average credit quality of the Libor contributors during the length of the tenor
  - Default events that could have been avoided, had the loan be made as rolling overnight
  - Survivorship bias that overnight loans benefit from
\[ U((r_t)) = \max_{(N_t)} U((r_t); (N_t)) \]
\[ V(L) = \max_N V(L; N) \]

- Best utilities a bank can achieve by lending OIS or Libor
- O/N process \( (r_t) \) assumed to be given depending on the supply/demand of liquidities and on base rates from the central bank

**LOIS equation**

\[ V(L^*) = U((r_t)) \]  \hspace{1cm} (3)

*Indifference value* at the optimal amounts prescribed by the solution to both optimization problems
Linear funding costs model

\[ \rho_t(D_t + x) = \alpha_t + \beta_t x \]  

Eg \( \alpha_t = 2\% \), \( \beta_t = 50\text{bp} \)

- The last euro borrowed by the bank was at an annualized interest charge of 2 cents
- If the bank would be indebted by €100 more, the next euro to be borrowed by the bank would be at an annualized interest charge of 2.5 cents
\[ T\mathcal{U}((r_t); (N_t)) = \mathbb{E} \int_0^T \left( (r_t - \alpha_t)N_t - \frac{1}{2} \beta_t N_t^2 \right) dt \quad (5) \]

Denoting by \( \lambda_t \) the (assumed) intensity of \( \tau \) and letting \( \gamma_t = \alpha_t + \lambda_t \) and \( \ell_t = e^{-\int_0^t \lambda_s ds} \), we also have:

\[ T\mathcal{V}(L; N) = \mathbb{E} \int_0^T \left( (L - \gamma_t)N - \frac{1}{2} \beta_t N^2 \right) \ell_t dt \quad (6) \]

where a standard credit risk computation was used to get rid of the default indicator.

- **Merton 74** A high-quality credit-name has a decreasing CDS curve reflecting the expected deterioration of his credit
- **Intensity** \( \lambda_t \) of \( \tau \) proxyed by the slope of the credit curve of a Libor representative (and therefore high-quality) borrower
  - Differential between the borrower’s 1y CDS spread and the spread of her short term certificate deposits
  - Currently 10 to a few tens of bp for major banks
  - **Credit skew** of a Libor representative borrower
Outline

1. Equilibrium Model

2. LOIS Formula


- $c_t := \alpha_t - r_t$ Funding spread

- OIS problem (1), (5) resolved independently at each date $t$

  
  $$
  u_t(r_t; N_t) = c_t N_t - \frac{1}{2} \beta_t N_t^2 \quad \leftarrow \quad \max N_t
  $$

  $$
  \rightarrow \quad U((r_t)) = U((r_t); (N^*_t)) = \mathbb{E} \left( \frac{1}{T} \int_0^T \frac{c_t^2}{2\beta_t} dt \right)
  $$

- Libor problem (2), (6) solved as

  $$
  T\mathcal{V}(L; N) = N\mathbb{E} \int_0^T (L - \gamma_t)\ell_t dt - \frac{1}{2} N^2 \mathbb{E} \int_0^T \beta_t \ell_t dt \quad \leftarrow \quad \max N
  $$

  $$
  \rightarrow \quad \mathcal{V}(L) = \mathcal{V}(L; N^*) = \left( \frac{\mathbb{E} \frac{1}{T} \int_0^T (L - \gamma_t)\ell_t dt}{2\mathbb{E} \frac{1}{T} \int_0^T \beta_t \ell_t dt} \right)^2
  $$
• Let $R := \mathbb{E} \frac{1}{T} \int_{0}^{T} r_t dt$
• Stylized LOIS defined as $(L^* - R)$, where $L^*$ is, given the process $(r_t)$, the solution to (3)
  • Assumed to exist, then unique and $\geq R$

**LOIS FORMULA**

\[ L^* - R \approx \lambda^* + \sigma^* \sqrt{T/2} \] (7)

• $\lambda^*$ Reference credit skew of the borrower
  • intrinsic value component of the LOIS
  • a borrower’s credit skew component

• $\sigma^*$ Reference volatility of the instantaneous funding spread process
  $c_t = \alpha_t - r_t$
  • assumed diffusive
    • factor $1/2$ under the square-root due to the fact that we deal with time-space variances
  • time-value of the LOIS
  • a lender’s liquidity (credit volatility) component
Outline

1. Equilibrium Model
2. LOIS Formula
Time series of the “red” intercepts (in %; credit component of the LOIS), “blue” and “purple” slopes (in %; liquidity component of the 3m- and 6m-LOIS) and “green” R2 coefficients of the regressions of the 1m to 1yr LOIS against $\sqrt{T/3m}$ or $\sqrt{T/6m}$
Three market regimes

- **Until Q1 of 2009**, the market seems to “try to understand” what happens, with an R2 becoming significant together with very large and volatile credit (“red intercept”) and liquidity (“blue or purple slope”) LOIS components.
  
  - Spike of both components at the turn of the credit crisis following [Lehman’s default in Sept 2008](#), during which the interbank drop of trust created both a credit and a liquidity crisis.

- **Between Q2 of 2009 and mid-2011**, the situation seems “stabilized” with an R2 close to 1, a liquidity LOIS component of the order of 30bps on the 3m or 45bps on the 6m and a much smaller credit LOIS component.

- **The ongoing Eurozone crisis, prompted by the US downgrade mid-2011**, reveals a third pattern with a much higher liquidity LOIS component, of the order of 60bps on the 3m or 90bps on the 6m, revealing increased funding liquidity concern of banks, due to harder regulatory constraints (e.g. government bonds no longer repositable).
14 Aug 2008

*Left* Euribor / EONIA-swap rates

*Right* Square root fit of the LOIS

\( T = 1\text{m to } 12\text{m} \)
28 Apr 2010

*Left* Euribor / EONIA-swap rates

*Right* Square root fit of the LOIS ($T = 1m$ to $12m$)
1 Dec 2011

*Left* Euribor / EONIA-swap rates

*Right* Square root fit of the LOIS

($T = 1$m to 12m)
Confrontation with the economical determinants of $\lambda_t$ and $\alpha_t$

- **Intercepts** of e.g. 10 bp appear as reasonable for a “credit skew”, that is, the differential between the one year CDS spread and the short term certificate deposit credit spread of a major bank.

- **Coefficients** of $\sqrt{T/2}$, ranging between 100 and 200 bp/yr (corresponding to magnifying on Fig. 19 by a factor 2 the purple curve, or slope coefficients of the regression against $\sqrt{T/6m}$), is quite in line with recent orders of magnitude of the volatility of major banks’ one year CDS spreads.
Theoretical developments corroborated by empirical evidence on the EUR market studied in this paper on the period half-2007 half-2012

LOIS explained in a balanced way by credit and liquidity until the beginning of 2009 and dominantly explained by liquidity since 2009
Implying the value $\sigma^*$ “priced” by the market from an observed LOIS ($L^* - R$) and a borrower’s CDS slope taken as a proxy for $\lambda^*$

- $\sigma^*$ can then be compared by a bank to an internal estimate of its 
  realized funding spread volatility, so that the bank can decide 
  whether it should rather lend Libor or OIS
- much like with going long or short an equity option depending on the 
  relative position of the implied and realized volatilities of the 
  underlying stock
- $\sigma^*$ can also be used for the calibration of the volatility $\sigma^*$ of the 
  funding spread process $c_t$ in a stochastic model for the latter
- e.g. in the context of multiple-curve CVA computations